The Nominal Theory of Interest under Habit Formation: Evidence for the U.S., 1959-2002.[*]

João Ricardo Faria[**]
School of Social Sciences, University of Texas at Dallas, Richardson, TX, USA.
André Varella Mollick[***]
Department of Economics, ITESM-Campus Monterrey, Monterrey, N.L., Mexico.

Abstract: This paper shows that habit formation in a three-period overlapping generations model leads to debt to money ratio and inflation affecting the nominal rate of interest. Employing monthly U.S. data from 1959:1 to 2002:2, we find that the rate of inflation leads to a strong and robust relationship in the long-run vector. The debt to money ratio (inflation rate) is negatively (positively) related to the interest rate in postwar U.S. data, consistent with habit formation and with the Fisher hypothesis, respectively. Vector autoregressions suggest a similar negative response of interest rate to shocks in the debt to money ratio.

Keywords: overlapping generations, nominal rate of interest, debt to money ratio, habit formation, cointegration.

JEL Classification: D91; E21.

---

[*] The authors are indebted to Marco A.C. Martins for bringing this topic to our research agenda and for several interesting discussions on this subject. We also would like to thank, without implicating, Francisco Carneiro and Miguel Leon-Ledesma for useful comments.
[**] Faria: School of Social Sciences, University of Texas at Dallas, P.O. Box 830688, GR 31, Richardson, TX 75083-0688, USA. Phone: 972-883 6402, Fax: 972-883 6297. E-mail: jocka@utdallas.edu
[***] **Corresponding Author**: Dr. André Varella Mollick, Department of Economics, DAF, ITESM-Campus Monterrey, E. Garza Sada 2501 Sur, 64849, Monterrey, N.L., Mexico. Phone: +52-81-8358-2000(ext. 4305, 4306), Fax: +52-81-8358-2000(ext. 4351). E-mail: avarella@itesm.mx

1. Introduction

One of the cornerstones of Monetary Economics, through the quantity theory of money (QTM), the Fisher equation establishes that a change in the quantity of money induces an equal change in nominal rates of interest [Lucas, (1980a)]. This relationship has been interpreted as supportive of the superneutrality of money hypothesis: an increase in inflation has no effect on real interest rates in the long run and, hence, on capital accumulation, output, employment and consumption as exemplified by the monetary growth model of Sidrauski (1967).

Several attempts to reexamine theoretically the nominal rate of interest have followed. Fried and Howitt (1983), by introducing bonds in Sidrauski (1967), show that inflation reduces the real interest rate on bonds, but leaves the marginal product of capital unaffected. This idea had been anticipated by Harrod (1971) and Roll (1972), and was fully developed by Martins (1980) who, in a three-period overlapping generations (OG) model, proposes a nominal theory of interest in which the only distinction between money and government bonds lies in the difference between their holding periods. The implied money demand only depends on the price level and income (given a constant velocity of circulation of money) and is not affected by the interest rate. The nominal rate of interest is determined by the relative supply of bonds with respect to money, bearing no direct relationship to the rate of inflation. One important consequence of this theory is that government bonds are net wealth, in contrast to the seminal result in Barro (1974).

Such unconventional results derive from the time structure and, in particular, from the transaction costs associated with money and bonds, which precludes bonds accumulated in any period to buy goods one period later. This raises the issue of the time structure of preferences, which are time-separable in Martins (1980), an assumption to be modified in the present paper for time non-separable preferences, such as habit formation. The explicit introduction of habit persistence in preferences implies that the nominal rate of interest becomes affected by the expected inflation rate, making the results closer to mainstream Monetary Economics.

Theory is not, however, the only motivation of this study. From an empirical perspective, the long-term relationship between interest rates and a ratio of debt to money is far from obvious in U.S. data. While not uniform across countries, empirical evidence is generally favorable to the rational expectations version of the Fisher hypothesis with constant ex-ante real rates [e.g., Engsted (1995)]. This implies that any theory that disregards inflation is difficult to reconcile with the data.

We apply this empirical observation to our central result. We find that, under habit formation, *inflation movements help determining the nominal rate of interest, together with the debt to money ratio*. In this sense, our model connects fundamental tenets of competing theories: debt from the fiscal theory and money from the QTM. See the portfolio choice perspective by Gordon and Leeper (2002) for similar motivations. In order to link the two theories to the Fisher hypothesis, we explore habit formation preferences.

The habit formation hypothesis has become widely used because it has been successful in explaining many important empirical problems ranging from consumption [Carroll and Weil (1994)] and its response to policy shocks [Fuhrer (2000)] or to exchange rate pegs [Uribe (2002)], to key features of business-cycle models [Boldrin et al. (2001)], and to the equity premium puzzle [Constantinides (1990)].

On monetary policy, in particular, the empirical tests in Fuhrer (2000) show that one can reject the hypothesis of no habit formation. The essence of the idea is that consumers wish to smooth both the level and the change in consumption. Thus, in response to shocks to interest rates or income, both the level and the change in consumption will respond gradually, leading to the hump-shaped response. The model without habit formation produces no such hump in the VAR, while simulations in Fuhrer (2000) show that the greater the importance of habit formation in the utility function, the more hump-shaped the response to an income shock.

None of the empirical literature on the neutrality of money has so far considered the empirical content of OG models for the nominal theory of interest.[1] In addition to non-stationary processes and structural breaks that often appear in the data, there is also one caveat that is perhaps responsible for shortcomings in the empirical front. According to Fisher's theory of interest, an exogenous rise in expected inflation for a certain horizon will produce an equivalent jump in the nominal yield on bonds of the same period. Since the results of estimating an equation in which the nominal rate depends on distributed lags of inflation expectations are poor, Sargent (1973) argues that the "negative empirical results carry no implications about the validity of the version of Fisher's theory considered here. … The theory cannot be tested by running regressions like Fisher's".

Applying the same logic, this paper does not claim to test the OG model in Martins (1980) through running reduced-form equations. Rather, this paper emphasizes that the

[1] On the neutrality of money, Fisher and Seater (1993) define as follows. For the *long-run neutrality* (LRN), permanent, exogenous changes to the level of money supply leave the level of real variables and the nominal rate of interest unchanged but ultimately lead to equiproportionate changes in the level of prices. For the *long-run superneutrality* (LRSN), permanent, exogenous changes to the growth rate of money supply ultimately lead to equal changes in the rate of interest, leaving the level of real variables unchanged. See also Sahu et al. (1990) on this issue and Noriega (2004) for recent international evidence.

long-run relationship, if any, that exists between the nominal rate of interest and the ratio of federal debt to money can only be valid *when inflation is introduced explicitly*. We find that the rate of interest, the ratio of federal debt to M1, and CPI inflation are cointegrated by Johansen methods. Eliminating inflation from the system, there is no such relationship. The estimated negative relationship between interest and debt to money is only possible to rationalize under habit formation. One possible interpretation is that, feeling that federal debt is getting higher, agents save more today, moving up the price of bonds and reducing the nominal rate of interest. This works as in Ricardian Equivalence propositions and encounters firm support in our model, matching reasonings presented on consumption dynamics by Fuhrer (2002) and Uribe (2002).

The estimated negative relationship is found both in the whole sample and once we take into account the structural break in U.S. interest rate of the early 80s, employing the Zivot and Andrews (1992) procedure to capture the more likely breakpoint. We next investigate vector autoregressions (VARs) that confirm negative, yet weak, responses of interest rates to shocks in the federal/debt money ratio. The caveat here is that the extended VAR can only be viewed as an approximation since the original equation is non-linear, a problem also found in models with discounting such as Canzoneri et al. (2001).

The rest of this paper is organized as follows. Section 2 presents the OG model with habit persistence. Section 3 gathers the data definitions and sources. Section 4 contains estimates of equations based on theoretical constructions with standard time separable preferences and with habit formation. Section 5 summarizes the conclusions of this study.

2. The Model

2.1. The demand for money and bonds

We consider the basic setup in Martins (1980). There is no capital and the only relevant difference between money and government bonds lies in their holding periods.[2] The government issues fiat money and two-period bonds. Individuals live for three periods, they are endowed with one unit of output (Y) in the first period of their lives, and there is no storage technology. They also receive in the initial period G units of nominal government transfer payments. They allocate their initial nominal income in goods (C), bonds (B), and money (M). Let $C_{i,j}$ stand for the ith period of life consumption and j indexes time. There are transaction costs involved in conversion of bonds into money before the retirement period, and individuals cannot use bonds accumulated in any period to buy goods one period later. They must accumulate money in period t to provide for consumption in period $t+1$. However, they can use both the money left over in period $t+1$ and the bonds accumulated in t to buy consumption in $t+2$. The budget constraint in each period of the life of the representative agent of generation t is the following:

$$P_t C_{1,t} + M_t + B_t [1 + i_t]^{-1} = P_t Y_t + G_t \tag{1}$$

$$P_{t+1} C_{2,t+1} + M_{t+1} = M_t \tag{2}$$

[2] Recent research on OG models that deal with the habit formation hypothesis focuses on the role of habit persistence on savings and its consequences for government real transfers. In a pure exchange economy, Lahiri and Puhakka (1998) show that habit persistence increases the desired savings of the young and, as a consequence, the government can sustain larger stationary deficits in the economy with habit persistence relative to an economy without habits. Along similar lines, Wendner (2002) shows that a rise in the strength of habits increases savings and capital accumulation. In relation to the optimal demand for money, the introduction of habit formation in the neoclassical monetary growth model by Faria (2001) leads to a more sensitive money demand than in Sidrauski (1967). In particular, money demand turns out to depend negatively on the nominal interest rate, which is in accordance with the widely accepted idea that money demand is basically affected by income and interest rate as in Lucas (1988).

$$P_{t+2}\,C_{3,t+2} = M_{t+1} + B_t \tag{3}$$

where P_j is the nominal price of consumption in period *j*, i_t is the two-period nominal rate of return on bonds. Therefore, $[1+i_t]^{-1}$ is the current nominal price of the government promise to pay one nominal unit worth of goods in period *t+2*.

Without loss of generality it is assumed that $Y_t = 1$. In addition, we restrict our attention to the case in which $i_t > 0$, since if $i_t < 0$, bonds are driven out of the economy, and in the case that $i_t = 0$, money and bonds are perfect substitutes. Finally, notice that, under $i_t > 0$, the individual uses only bonds to provide for consumption in period *t+2*. As a consequence: $M_{t+1} = 0$. The transaction costs imply that in midlife the agent is subject to a Clower constraint as in Lucas (1980b).

The utility function *U* is defined over non-negative consumption during the first, second and third periods of life. It is twice continuously differentiable, increasing in all arguments, and strictly concave. In the present case, the use of habit persistence implies that individuals wish to smooth their consumption pattern, minimizing changes in consumption. That is, higher first and second period consumption reduces the utility derived from a given level of the third period consumption, such as: $Z = C_{3,t+2} - \delta_2\, C_{2,t+1} - \delta_1\, C_{1,t} > 0$, where δ_i is the habit persistence parameter associated with the ith period, assumed to be positive but less than unity. The utility function studied in this paper has the following form[3]:

[3] The individual derives utility not only from the absolute level of consumption in the three periods but also from increasing consumption in the third period relative to the first two periods. In this formulation agents

$$U_t = \log C_{1,t} + \log C_{2,t+1} + \log Z \tag{4}$$

The economically meaningful area is given by downward sloping indifferences curves:

$$\frac{dC_{3,t+2}}{dC_{2,t+1}} < 0; \frac{dC_{3,t+2}}{dC_{1,t}} < 0; \frac{dC_{2,t+1}}{dC_{1,t}} < 0$$

Therefore it is necessary to assume that: $C_{3,t+2} - \delta_2 C_{2,t+1} > 2\delta_1 C_{1,t}$ and $C_{3,t+2} - \delta_1 C_{1,t} > 2\delta_2 C_{2,t+1}$. The consumer problem of the representative agent of generation t is to maximize (4) with respect to M_t and B_t subject to the budget constraints (1)-(3). The first order conditions are the following:

$$\frac{-1}{P_t C_{1,t}} + \frac{1}{M_t} - \left[\frac{\delta_2 / P_{t+1} + \delta_1 / P_t}{Z}\right] = 0 \tag{5}$$

$$\frac{-[1+i_t]^{-1}}{P_t C_{1,t}} + \left[\frac{1/P_{t+2} - \delta_1 [1+i_t]^{-1} / P_t}{Z}\right] = 0 \tag{6}$$

Equations (5) and (6) together with (1)-(3) yield the solution to the consumer problem determining the consumption plan $(C_{3,t+2}; C_{2,t+1}; C_{1,t})$ and the demands for M_t and B_t .

---

take more than one period of time to adjust their habits, which allows much needed mathematical simplification. See Araújo and Martins (1999) for extensions of the OG model in Martins (1980).

Of particular interest here are the demand functions for M_t , B_t , and $C_{1,t}$:

$$P_t C_{1,t} = M_t A \tag{7}$$

$$M_t = \frac{P_t + G_t}{1 + A + \Theta} \tag{8}$$

$$B_t = (1 + i_t)\Theta M_t \tag{9},$$

where:

$$A \equiv \frac{1 + i_t + (1 + \pi_{t+1})\delta_2}{1 + i_t + (1 + \pi_{t+1})\,(1 + \pi_t)\delta_1}$$

$$\Theta \equiv A\left[1 + \frac{2(1 + \pi_{t+1})\,(1 + \pi_t)\delta_1}{1 + i_t}\right] + \delta_2 \frac{(1 + \pi_{t+1})}{1 + i_t}$$

$$(1 + \pi_{t+1}) \equiv \frac{P_{t+2}}{P_{t+1}}$$

The term π_t denotes the expected inflation rate of period *t*. It is worth noticing that A and Θ are functions of habit persistence parameters and are non-linear functions of the nominal interest rate, and expected inflation rate. See the Appendix.

As expected, when there is no habit persistence: $\delta_1 = \delta_2 = 0$, it follows that $A = \Theta = 1$, and the model collapses to Martins (1980) case. It is easy to see that when δ_1 and/or δ_2 are different from zero the implications of the model cannot be more distinct from Martins (1980), since with habit persistence the individual takes into account the real opportunity cost of his decisions regarding the allocation of his nominal income. This happens because, with habit persistence, higher levels of consumption in the first and second periods reduce the utility derived from a given level of third period consumption. In this sense, habit persistence leads the individual to foresee and anticipate the impacts of his current and near future consumption decisions in the last period of his life.

The present model differs from Martins (1980) in three important issues. First, notice that by equation (8) the demand for money now becomes sensitive to the nominal interest rate and to the expected inflation. Second, in the OG model with time separable preferences the individual allocates exactly 1/3 of his nominal income in goods, money and bonds in the first period of his life. With habit persistence, however, the individual takes into account the nominal interest rate as well as the expected future inflation in order to demand bonds, goods and money.

A simple example can give us an idea of how the individual allocates his nominal income. On the one hand, by assuming that $(1+\pi_t)\delta_1 < \delta_2$, it implies that A > 1 and $\Theta > 1$. As a consequence, in period *t*, the individual allocates less than 1/3 of his nominal income in money and, therefore, consume more of the consumption good and/or bonds than in Martins (1980) model. On the other hand if $(1+\pi_t)\delta_1 > \delta_2$, it implies that A < 1, and the individual decides to allocate a smaller part of his nominal income in consumption goods, therefore increasing the consumption of money and bonds.

Thirdly, observe the demand for bonds. Notice that, from (9) we have:

$$i_t = \frac{B_t}{\Theta M_t} - 1 \tag{9'}$$

In Martins (1980) nominal interest is basically determined by the bonds/money ratio, since $\Theta = 1$. From (9') one can see that the nominal rate of interest depends on Θ, which implies that it is affected not only by the bonds/money ratio, but also by the expected inflation rate. This makes (9') one of the so-called Fisher equations by Azariadis (1993).

In the original theory of nominal interest rate determination, the government can give a real transfer payment to the generation that is old in period *t+2* by doubling B_t and keeping M_t constant. Due to the presence of transaction costs individuals cannot convert one two-period bond into two one-period bonds and, consequently, they cannot consume the initial increase of wealth [due to rise of B_t] over all three periods of life and are constrained to consume it all in the last period. As future money supply is unchanged, P_{t+2} does not double, which leads to a rise in $C_{3,t+2}$. As a result, in Martins (1980), government bonds are net wealth, in contrast to Ricardian equivalence [Barro (1974)].

With habit persistence, however, this is no longer true. Despite the complex non-linear relationship that determines the nominal interest rate in (9'), which may lead to multiple equilibria, we can try to make the point as clear as possible. Assume that $\Theta > 1$, for a sufficiently high Θ, from (9') the price of bonds (the rate of interest) more than doubles (falls more) when the government doubles B_t . This may lead to an increase in the demand for money and/or goods, increasing the price level. One cannot rule out the possibility that P_{t+2} can increase by more than the double, leading to a fall in consumption in period t+2. Provided one keeps in mind that this analysis does not account for changes in the nominal rate of interest and/or in the expected inflation on money demand, the consequence is clear: *bonds are not necessarily net wealth*.

2.2 General equilibrium

In order to close the model it is necessary to consider the supply side of the economy. As there is no production, there are no firms and the supply of money and bonds are both determined by the government budget constraint. Government revenue is $M_t - M_{t-1} + B_t[1+i_t]^{-1}$, since the member of generation *t* wishes to accumulate M_t units of money and B_t units of bonds at the nominal price of $[1+i_t]^{-1}$, and the member of *t-1* generation wishes to exchange M_{t-1} nominal units of money for goods. Government spending is $G_t + B_{t-2}$, since the member of generation *t-2* wishes to retire B_{t-2} nominal units of bonds. The supply side of the economy is given by:

$$G_t + B_{t-2} = M_t - M_{t-1} + B_t[1+i_t]^{-1} \tag{10}$$

The general equilibrium model is composed by the system of equations (7)-(10). As the values of B_{t-2} and M_{t-1} are given at the start of period *t*, one ought to consider that the government controls the variables G_t and B_t. From (8) and (10):

$$M_t = [G_t + B_{t-2} + M_{t-1}][1+\Theta]^{-1} \tag{11}$$

This solution to M_t is completely independent from the current stock of bonds B_t. In Martins (1980) money finances half the transfer payments. In the present setup, however, money can actually finance more than half the transfer payments if $\Theta > 1$. The determination of the current price level follows from (11) and (8):

$$P_t = AM_t + M_{t-1} + B_{t-2} \tag{12}$$

The current price level is given by the history of stock of government securities over three periods and is also impacted by inflationary expectations and by the level of nominal interest rate through the term *A*. The equilibrium amount of current consumption goods is found using equations (7), (11) and (12):

$$C_{1,t} = \frac{[G_t + B_{t-2} + M_{t-1}][1+\Theta]^{-1}}{AM_t + M_{t-1} + B_{t-2}} A \tag{13}$$

Finally, the market nominal rate of interest is found by inserting (12) into (9):

$$i_t = \frac{AB_t}{\Theta[P_t - M_{t-1} - B_{t-2}]} - 1 \tag{14}$$

According to (14), the nominal rate of interest is related to the price level, now bearing a strong relationship to the expected rate of inflation as in the Fisher equation.

3. The Data

All data come directly from the Federal Reserve Bank of Saint Louis – FRED Economic Data, currently available at (http://research.stlouisfed.org) website. The basic debt variable is the Debt of Domestic Nonfinancial Sectors – Federal (we call it FD), Seasonally Adjusted, in Billions of Dollars, Source: H.6 Release - Federal Reserve Board (FRB) of Governors. For the alternative debt variable, we use the Debt of Domestic Nonfinancial Sectors (call it D) - Outstanding Credit Market Debt of U.S. Government, State and Local Governments, and Private Nonfinancial Sectors, Seasonally Adjusted, in Billions of Dollars, Source: H.6 Release – FRB of Governors.

Monetary aggregates include M1, M2, and M3 Money Stock, Seasonally Adjusted, in Billions of Dollars, Source: H.6 Release - FRB of Governors. The interest rate variable is the 3-Month Treasury Bill Rate (we call it TB3), Secondary Market, Averages of Business Days, Discount Basis, in percent, Source: H.15 Release - FRB of Governors. The Consumer Price Index for All Urban Consumers: All Items, 1982-84=100, is Seasonally Adjusted, Source: U.S. Department of Labor, Bureau of Labor Statistics. We divide either FD or D by each money stock and obtain the series FDM1, FDM2, FDM3, DM1, DM2, and DM3. For reasons discussed later on, we will stick to FDM1 as the empirical counterpart of the theoretical debt to money ratio.

During the summer of 2002, a restructuring of the data set on fiscal series changed slightly the definition of the federal debt data. As stated, this paper uses Debt of Domestic Nonfinancial Sectors – Federal (1959:1 to 2002:2), which is now available under the title “Federal Government Debt – Net”, seasonally adjusted, end of month totals in US$ billion. All other data have remained unchanged.

4. Empirical Results

4.1. Preliminaries

It is important to understand the reasons why the order of the series matters in this context. First, as argued in Fisher and Seater (1993), in order for inferences regarding LRN (LRSN) to be drawn from a reduced form, the data must contain permanent stochastic changes in levels (or in the growth rate) of the series (of money supply if we are dealing with LRN and LRSN). Second, the potential long-run response of one variable to another depends on their relative orders of integration. Recent international evidence in Noriega (2004) shows that it is generally difficult to arrive at conclusive results regarding the order of integration of money aggregates on the basis of three typical unit root tests.

Consider first the unit root evidence in table 1 for the period 1959:01 to 2002:02. Since unit root tests have admittedly low power in finite samples, three types of tests are used. The first column contains the usual Augmented Dickey-Fuller (ADF) test. We employ the sequential approach suggested by Ng and Perron (1995) for choosing the number of lags in the estimating equations that appear in parenthesis close to the reported ADF figure. We also employ the Elliott-Rothenberg-Stock (ERS) generalized least squared test in column 2, which is particularly useful when the series has an unknown mean or linear trend. Both ADF and ERS tests are computed under the null that the series is non-stationary. Finally, in column 3 we report the results of the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, in which the null hypothesis becomes the stationary series.

[Table 1 here]

All these tests in the upper part of table 1 show that, when a broader monetary aggregate than M1 is used the debt to money ratio (FDM2 and FDM3) series do not achieve stationarity in first-differences. For this reason, we focus on the FDM1 ratio henceforth. As can be seen in table 1, the results are consistent across the three tests. For example, for the federal debt to money ratio (FDM1), the ADF and ERS tests reject the null of non-stationarity of the series in first-differences at 1% and 5%, respectively. The KPSS in this case rejects the null of stationarity in levels at 1% and does not reject the null in first-differences, which again suggests an integrated of order 1 process [I (1)] for FDM1.

The rate of inflation, represented by ΔP, obeys a similar pattern except for the rejection at 10% of the null of the series in levels at the ADF. Since the ERS test and KPSS are compatible with a I(1) process for the rate of inflation at standard confidence levels, we disregard the rejection in levels at 10% given by the ADF test. We thus take the inflation series as I(1). This also happens in the interest rate data (TB3) since the null in levels is also rejected at 10%, which is also observed under the ERS test. However, the apparent stationarity of the interest rate series in levels is not present under the KPSS, which supports an I(1) pattern instead. Since the levels for rejection in ADF and ERS are of 10%, we prefer to follow the KPSS result, while noting the borderline result of the Dickey-Fuller type tests for the rate of interest.

The general conclusion that follows from table 1 is that there appears to be non-stationarity in levels. However, the possibility of break in the series has to be considered when there are abnormal shifts in interest rates as is clear from inspection of figure 1. The unit root may reflect a temporary shift due to a change in monetary regime. It is known that the 1979-1980 period was characterized by a change in monetary policy in the U.S. Specifically, the FED ended targeting nominal interest rates in 1979:9 and, from 1979:10 to 1982:9, used money base control, allowing interest rates to fluctuate freely. Thereafter, the FED returned to a system of targeting nominal rates. Consequently, a regime shift may reduce the power of the ADF test by biasing it towards non-rejection of the null, which requires investigation including a dummy variable.

In order to take into account the recent literature on interest rates and also to reduce computational burden, we focus on the 1979-1980 period. Focusing on the nominal interest rate series individually, we estimate model A as in Zivot and Andrews (1992) and obtain May of 1980 as the minimizer of the t-values on the relevant regressor of the modified ADF test with break.[4] The null has the process as structurally stable with unit root and drift and the alternative hypothesis has the process as trend stationary with a one time shift in its mean at an unknown date. Our results in the bottom part of table 1 show that, allowing for structural change does not make any of the series stationary in levels. According to either model A or B as in Zivot and Andrews (1992) it is still not possible to reject the null.

4.2. Main Results

---

[4] We get t=-1.682 for model A (shift level model) with the dummy set at May of 1980. With the dummy set at April of 1980, we get t = -1.662 and at June of 1980 we obtain get t = -1.634. Values are much higher for dummy variables set at time periods more distant from May of 1980. This coincides exactly with the breakpoint estimated by Tzavalis and Wickens (1996) over the period 1970:1 to 1991:02. A similar result is obtained for model B (growth shift model), since we obtain t=-1.771 for the model with the dummy set at May of 1980, while we get t = -1.737 and at April of 1980 and we obtain t = -1.664 for June. Values are also higher for dummy variables set at time periods more distant from May of 1980. By the Inf ADF test, we thus select May of 1980 according to both ways of modeling structural change in the 3-month nominal interest rate.

As a first pass to long-run estimation, standard OLS estimates of versions of the basic and modified models are presented in table 2. The estimated value of β_1, the coefficient of debt/money on interest rates, varies from –4.0 to –5.80 for the models with dummy and time trend. These values suggest that, the higher debt/money ratio, the lower is the nominal interest rate, in sharp contrast to the basic model in Martins (1980). Note that, under habit formation, a higher debt/money ratio does not necessarily lead to a higher interest rate. This is so because we may have $A > 1$ and thus $\Theta > 1$, which may explain the negative relationship, as discussed next to (9').

[Table 2 here]

A battery of diagnostic tests on the residuals, however, suggests serious misspecifications in linear estimations such as these. We find evidence of serial correlation, heteroscedasticity and non-normality applying standard residual tests on the models in table 2. Besides, the evidence from table 1 is more consistent with covariance non-stationary processes, which invalidate the preceding statistical inferences.

After the I(1) judgement made on the three series and leaving the OLS evidence in table 2 aside, we next perform Johansen cointegration tests. The first set of tests stands for the standard OG model with time separable preferences. That is, the specification that parallels the one developed in Martins (1980) without habit persistence that yields the nominal interest rate as determined entirely by the bonds/money ratio ($\Theta = 1$ in equation (9')).

If one believes that all trends are stochastic, the intercept is included in cointegrating equation and test VAR. If, however, some of the series are taken as trend stationary, the intercept and trend are included in cointegrating equation and no trend is included in test VAR. We prefer to suppose that all trends are stochastic and reproduce in table 3 the set of estimates when all trends are taken as stochastic. Lag length systems of 5, 7, and 8 are chosen by various criteria as pointed out in the table.[5]

[Table 3 here]

As can be seen in the upper part of table 3 the 8.57 value of the trace test is lower than the 15.41 critical 5% level, thus not rejecting the null of no cointegrating equation. The same holds for the maximal eigenvalue test, computed at 6.79. From this evidence, it appears that the nominal rate of interest bears no long-run relationship with the federal debt/money ratio.

One may suspect, based on the Fisher equation, that inflation is an essential component of the nominal theory of interest. We thus explore a bivariate VAR with only interest rate and inflation. It is now possible to reject the null of no-cointegration according to both trace and maximal eigenvalue tests. Normalizing the 3-month interest rate to one, a statistically significant coefficient is estimated at +19.3 for the rate of inflation by the Johansen method.

Consider next the specification consistent with the habit formation model. The vector of variables is [TB3, FDM1, ΔP] and yields only one cointegrating vector with statistically significant coefficients –1.90 for the FDM1 and 16.7 for ΔP. This supports a negative relationship between the debt/money ratio and the rate of interest. The relationship is positive according to the standard theory of interest in Martins (1980) but

[5] In order to see the change in the results, we estimate all models under the assumption that some of the series are taken as trend-stationary. The results are generally weaker than those reported below for the case of stochastic trend. In particular, the coefficients on the debt/money ratio are not statistically significants, which does not support the OG models discussed above. Since we believe that the series are more likely to follow stochastic trends, we feel that the estimates of table 3 are the best among the possible choices in the computational routine of the Johansen procedure.

may be easily rationalized to be negative in the context of habit formation. As mentioned, feeling that federal debt is getting higher, agents save more today, moving up the price of bonds and reducing the nominal rate of interest. Similar reasonings have been used by Fuhrer (2002) and Uribe (2002) to rationalize consumption responses to income shocks or to exchange rate pegs, respectively.

Finally, in order to consider the literature on structural break, we reestimate the VARs of the Johansen procedure including the dummy variable of May of 1980, the breakpoint obtained by search under the Inf ADF test developed by Zivot and Andrews (1992). The basic results are maintained: in the extended model [TB3, FDM1, ΔP, dum8005] there is definitely cointegration according to both trace and maximal eigenvalue tests, yielding coefficients on the rate of inflation of +17.8 and on debt/money of –3.37. Again both coefficients are statistically significant and can be easily justified theoretically. The model without inflation, however, fails to reject the null under the maximal eigenvalue test at 1%, suggesting the long-run relationship is not so firm when ΔP is left out of the picture.

Turning now to dynamic responses, the issue of causality among the variables needs to be addressed. We perform pairwise Granger causality tests on the three variables of interest. The more general specification involves the system [ΔP, FDM1, TB3], estimated with 5 lags (N=513) selected by likelihood ratio, final prediction error, and Akaike criterion.[6] The more restricted specification [FDM1, TB3] compatible with the OG model with standard preferences is estimated with 7 lags (N=515) selected by likelihood ratio, final prediction error, Akaike and Hannan-Quinn criteria. The bivariate

[6] The Hannan-Quinn criterion selected 4 lags and the Schwarz 3 lags. The results are not sensitive to these particular lags instead of the adopted 5 lags.

VAR is straightforward but note that the relationship among the inflation rate, debt to money and the interest rate is very complicated from (9') or (14). It is non-linear and therefore the VAR needs to be seen as an approximation[7]

The null hypothesis that inflation rate does not cause FDM1 is rejected with F-statistic = 6.47 [p-value = 0], while FDM1 does not cause inflation in Granger's sense [F stat. = 1.35; p-value = 0.24]. There appears to be bidirectionality between the inflation rate and nominal interest rate: F-statistic for TB3 causing prices = 7.55 and F-statistic for prices causing TB3 = 3.42 [p-values = 0 in both cases]. However, troubling for direct tests of equations such as (9') is the relationship between the rate of interest and FDM1, since the hypothesis that TB3 causes FDM1 is rejected with F-statistic = 5.15 [p-value = 0], while the hypothesis that FDM1 causes inflation [F stat. = 0.54; p-value = 0.75] can not be rejected.

We perform several VAR estimations of bivariate and trivariate systems. Again, our model implies a non-linear relationship and the extended VAR must be taken as an approximation. Since, from a statistical point of view, the direction of causality is reversed to what we obtain from the theory in (9') or (14), we estimate the VARs under various Cholesky orderings and our preferred system is chosen by economic theory. In other words, the ordering for Cholesky obeys [FDM1, TB3] and [ΔP, FDM1, TB3], although we investigate alternative orderings.[8]

---

[7] The nonlinearity arises in our model due to habit formation. But it may derive in other contexts from discounting, such as in the theory of the role of fiscal policy in price determination. For example, Canzoneri et al. (2001) derive a theoretical relationship for government surpluses, liabilities, and the discount factor. Their VAR analysis is bivariate at first and then extended to the discount factor. Here, we have no way to circumvent the nonlinearity of the model and appeal to the approximation of the VAR.

[8] For example, an alternative would be to let the Wold criterion of more exogenous to less exogenous in the variance decompositions determine our choice of the preferred system. However, the results change only in very few cases, such as in the bivariate system (in both levels and first-differences) when we get negative responses of interest rates to FDM1 that is turned flat under the alternative (statistic) criterion. Apart from

Consider first the bivariate system [FDM1, TB3], with the Cholesky decomposition set in such order, since this is the only ordering that makes sense in both Martins (1980) model and our modified model for habit formation. In the upper part of figure 2, for the system in levels, we see that the nominal interest rate (TB3) falls on impact, lasting until 3 months, due to a rise in FDM1. The 5% confidence bands suggest the reponse turns to zero after the 4^{th} month, but the negative response in the first 3 months is easily rationalized under habit formation, as explained in section 2. Note the explosive behavior of FDM1 in the long run to its own shocks. In contrast, observe the mean reverting pattern for the nominal interest rate: the impact on the 3-month rate is almost 0.5% to a 1% shock, moving up to 0.6% and then stabilizing at 0.3% in the 2-year horizon. For the system in first-differences in the bottom, interest rates also respond negatively on impact to higher standard deviations of the debt to money ratio, although turn to zero quickly as is often observed in differenced-VAR models.

Turning to the trivariate and approximate VAR to the mathematical relationship in (9') or (14), we report in figure 3 the last row of the VAR with Cholesky orderings based on economic theory: [ΔP, FDM1, TB3]. In the upper panel of figure 3, the response in levels of interest to a rise in the shocks to debt to money is again negative on impact: less than -0.1% due to a shock of 1%. Higher shocks to inflation lead to a higher and persistent nominal interest rate, as predicted by Fisher's hypothesis, while the nominal interest rate is mean-reverting to shocks of its own. In the lower panel of figure 3, one sees in the middle chart the negative response of interest rate to positive shocks in debt to money ratio: again a close to –0.1% response. This paper, assuming agents account also

---

this, there are no other noticeable changes with respect to our benchmark chosen by economic theory considerations.

for *changes in consumption*, is consistent with forward looking agents that change consumption spending plans in order to minimize consumption changes.[9]

5. Concluding Remarks

This paper introduces habit formation in Martins (1980) three-period OG model, modifying entirely the nominal theory of interest. The model generates a Fisher-like equation in which the expected inflation rate affects the nominal rate of interest, a proposition firmly grounded in postwar U.S. data. We first employ Johansen cointegration methods to non-stationary processes, moving next to VARs and dynamic responses in the nominal interest rate to shocks in the debt to money ratio. The caveat is that, under habit formation, there is a non-linear relationship and the extended VAR can only be taken as an approximation. The bivariate VAR, however, yields a negative relationship between the nominal interest rate and debt to money ratio.

By assuming that agents account not only for the levels of consumption but also for *changes in consumption*, the model implies forward looking agents, who alter their consumption spending decisions in order to minimize consumption changes. This pattern seems to prevail in consumption research as well [e.g., Fuhrer (2000) and Uribe (2002)]. In this article, habit formation transforms the non-orthodox model in Martins (1980) into a setting in which Ricardian equivalence holds. Recent evidence for postwar U.S. in Canzoneri et al. (2001) shows that Ricardian regimes are more empirically plausible than

[9] Uribe (2002), for example, discusses the price-consumption puzzle within exchange rate-based stabilization programs. This takes the form, in his model, of a once-and-for-all reduction in the rate of currency devaluation, which implies a permanent decline in interest rates and a positive wealth effect. Under time separable preferences, the positive wealth effect induces a once-and-for-all increase in spending in tradables and non-tradables. Under habit formation, however, such consumption boom is not optimal because the increase in habit that results from higher consumption would make future marginal utilities of consumption larger than the current, with agents shifting consumption into the future.

non-Ricardian regimes. Since Ricardian regimes are at least as theoretically plausible than non-Ricardian regimes, a case may be made for the former. Similarly, this paper brings the OG model closer to Ricardian equivalence and to empirical features of postwar U.S. data, such as the incorporation of inflation to the theory of nominal interest. Doing so, our model links the fiscal theory to the QTM [Gordon and Leeper (2002)].

Appendix

These are the effects of inflation rate, habit persistence and nominal interest rate on functions A and Θ :

$$\frac{\partial A}{\partial \delta_2} > 0; \frac{\partial A}{\partial \delta_1} < 0; \frac{\partial A}{\partial \pi_t} < 0;$$

$$\frac{\partial A}{\partial \pi_{t+1}} \begin{smallmatrix} \geq \\ < \end{smallmatrix} 0 \Leftrightarrow (1+\pi_t)\delta_1 \begin{smallmatrix} < \\ \geq \end{smallmatrix} \delta_2$$

$$\frac{\partial A}{\partial i_t} \begin{smallmatrix} \geq \\ < \end{smallmatrix} 0 \Leftrightarrow (1+\pi_t)\delta_1 \begin{smallmatrix} \geq \\ < \end{smallmatrix} \delta_2$$

$$\frac{\partial \Theta}{\partial \delta_2} = \frac{\partial A}{\partial \delta_2}\left[1 + \frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{1+i_t}\right] + \frac{(1+\pi_{t+1})}{1+i_t} > 0$$

$$\frac{\partial \Theta}{\partial \delta_1} = \frac{\partial A}{\partial \delta_1}\left[1 + \frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{1+i_t}\right] + A\frac{2(1+\pi_{t+1})\,(1+\pi_t)}{1+i_t}$$

$$\frac{\partial \Theta}{\partial \pi_t} = \frac{\partial A}{\partial \pi_t}\left[1 + \frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{1+i_t}\right] + A\frac{2(1+\pi_{t+1})\,\delta_1}{1+i_t}$$

$$\frac{\partial \Theta}{\partial \pi_{t+1}} = \frac{\partial A}{\partial \pi_{t+1}}\left[1 + \frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{1+i_t}\right] + A\frac{2\,(1+\pi_t)\delta_1}{1+i_t} + \frac{\delta_2}{1+i_t}$$

$$\frac{\partial \Theta}{\partial i_t} = \frac{\partial A}{\partial i_t}\left[1 + \frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{1+i_t}\right] - A\frac{2(1+\pi_{t+1})\,(1+\pi_t)\delta_1}{[1+i_t]^2} - \delta_2\frac{(1+\pi_{t+1})}{[1+i_t]^2}$$

References

Araújo, Jorge T. and Martins, Marco Antonio C. (1999) Economic growth with finite lifetimes. *Economics Letters* 62, 377-381.

Azariadis, Costas (1993) *Intertemporal Macroeconomics*. Oxford: Blackwell.

Barro, Robert J. (1974) Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1117.

Boldrin, Michele, Christiano, Lawrence and Fisher, Jonas (2001) Habit persistence, asset returns, and the business cycle. *American Economic Review* 91 (1), 149-166.

Canzoneri, Matthew, Cumby, Robert and Behzad Diba (2001) Is the price level determined by the needs of fiscal solvency? *American Economic Review* 91 (5), 1221-1238.

Carroll, Christopher D. and Weil, David N. (1994) Saving and growth: A reinterpretation. *Carnegie-Rochester Conference Series on Public Policy* 40, 133-192.

Constantinides, Georgios M. (1990) Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519-543.

Engsted, Tom (1995) Does the long-term interest rate predict future inflation? A multi-country analysis, *Review* of *Economic Studies* 77, 42-54.

Faria, João R. (2001) Habit formation in a monetary growth model. *Economics Letters* 73, 51-55.

Fisher, Mark and Seater, John (1993) Long-run neutrality and superneutrality in an ARIMA framework. *American Economic Review* 83 (3), 402-415.

Fried, J. and Howitt, Peter (1983) The effects of inflation on real interest rates. *American Economic Review* 73, 968-980.

Fuhrer, Jeffrey C. (2000) Habit formation in consumption and its implications for monetary-policy models. *American Economic Review* 90, 367-390.

Gordon, Robert and Leeper, Eric (2002) The price level, the quantity theory of money, and the fiscal theory of the price level. *NBER Working Paper* 9084, July. http://papers.nber.org/papers/w9084.pdf

Gregory, Alan and Hansen, Bruce (1996) Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics* 70, 99-126.

Harrod, R. F. (1971). Comments (on paper by Milton Friedman). In: G. Clayton et al. (eds.), *Monetary Theory and Monetary Policy in the 1970's*. London: Oxford University Press.

Lucas, Robert E., Jr. (1980a) Two illustrations of the quantity theory of money. *American Economic Review* 70, 1005-1014.

Lucas, Robert E., Jr. (1980b) Equilibrium in a pure currency economy. *Economic Inquiry* 18, 203-220.

Lucas, Robert E., Jr. (1988) Money demand in the United States: A quantitative Review. *Carnegie-Rochester Conference Series on Public Policy* 29, 137-168.

Lahiri, A. and Puhakka, M. (1998) Habit persistence in overlapping generations economies under pure exchange. *Journal of Economic Theory* 78, 176-186.

Martins, Marco Antonio C. (1980) A nominal theory of the nominal rate of interest and the price level. *Journal of Political Economy* 88, 174-185.

Ng, Serena and Perron, Pierre (1995) Unit root test in ARMA models with data dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association* 90, 268-281.

Noriega, Antonio (2004) Long-run monetary neutrality and the unit root hypothesis: further international evidence. *The North American Journal of Economics and Finance*, forthcoming.

Roll, Richard (1972) Interest rates on monetary assets and commodity price index changes. *Journal of Finance* 27, 251-277.

Sidrauski, Miguel (1967) Rational choice and patterns of growth in a monetary economy. *American Economic Review Papers and Proceedings* 57, 534-544.

Sahu, A. P., Jha, R. and Meyer, L. H. (1990) The Fisher equation controversy: A reconciliation of contradictory results. *Southern Economic Journal* 57, 106-113.

Sargent, Thomas (1973) Rational expectations, the real rate of interest, and the natural rate of unemployment. *Brookings Papers on Economic Activity* 2. Reprinted in: Lucas Jr. R. and Sargent, T. (eds.) (1981), Rational Expectations and Econometric Practice. London: George Allen & Unwin, 159-198.

Tzavalis, Elias and Wickens, M. R. (1996) Forecasting inflation from the term structure. *Journal of Empirical Finance* 3, 103-122.

Uribe, Martin (2002) The price-consumption puzzle of currency pegs. *Journal of Monetary Economics* 49, 533-569.

Wendner, R. (2002) Capital accumulation and habit formation. *Economics Bulletin* 4 (7), 1-10.

Zivot, Eric and Andrews, David (1992) Further evidence on the Great Crash, the oil-price shock, and the unit-root hypothesis. *Journal of Business & Economic Statistics* 10 (3), 251-270.

Table 1. Unit Root Tests on Key Variables: 1959:1 to 2002:2.

		Standard Unit Root Tests					
		ADF (k)		**ERS**		**KPSS**	
Series	**Trend?**	**Levels**	**First Diffs.**	**Levels**	**First Diffs.**	**Levels**	**First Diffs.**
FDM1	Yes, up	-2.20(8)	-3.62(7)***	-1.52(8)	-3.34(7)**	0.54 (17)***	0.31(15)
FDM2	No	-2.54(17)	-2.27(12)	-1.51(14)	-1.14(13)**	1.27 (17)***	0.54(16)**
FDM3	No	-2.82(14)*	-2.05(13)	-0.86(15)	-1.15(14)	0.61 (17)**	0.51(17)**
TB3	No	-2.68(16)*	-4.65(18)***	-1.76(16)*	-3.69(15)***	0.54 (17)**	0.03(3)
ΔP	No	-2. 64(13)*	-8.28(12)***	-1.25(14)	-8.28(13)***	1.24 (16)***	0.20(227)

		Unit Root Tests with Structural Break (in May of 1980) Zivot and Andrews (1992)			
Series	**Test?**	**Model A**		**Model B**	
		Levels	**First Diffs.**	**Levels**	**First Diffs.**
Fixed breakpoint					
FDM1	ADF(k)	-2.39(8)	-3.59(7)*	-2.60(8)	-3.60(7)
ΔP	ADF(k)	-2.49(12)	-8.46(12)***	-3.02(12)	-8.62(12)***
Selected breakpoint					
TB3	Inf ADF	-1.68(12)	-5.89(12)***	-1.77(11)	-5.91(12)***

Notes: Monthly data are used as explained in the data section. The variables are defined as follows: **FDM1** stands for the federal debt to M1 ratio; **FDM2** stands for the federal debt to M2 ratio; **FDM3** stands for the federal debt to M3 ratio; **TB3** stands for the three months Treasury bill constant maturity rate; and **ΔP** stands for the rate of inflation gauged by the variation month to month of the consumer price index (CPI). ADF (k) refers to the Augmented Dickey-Fuller t-tests for unit roots. For the series in levels in the upper part of the table, the ADF (k) of each entry is placed with a constant and trend or just a constant depending on what is suggested by visual plots. The lag length (k) is chosen by the Campbell-Perron data dependent procedure, whose method is usually superior to a fixed k chosen a priori and to k chosen by the information criterion. See Ng and Perron (1995). The method starts with an upper bound, k_{max}=18, on k. If the last included lag is significant, choose k = k_{max}. If not, reduce k by one until the last lag becomes significant (we use the 5% value of the asymptotic normal distribution to assess significance of the last lag). If no lags are significant, then set k = 0. Next to the ADF t-value, in parenthesis is the selected lag length. For the ERS Dickey Fuller-Generalized Least Squares test, the maximum number of lags is 18 and the selected one (in parenthesis) is the one chosen by Akaike Information Criterion. For the KPSS test, the spectral estimation method uses the Bartlett kernel and bandwidth (in parenthesis) is given by Newey-West procedures. For the ADF and ERS tests, the symbol * attached to the figure indicates rejection of the null of no-stationarity at the 10% level. The symbol ** indicates rejection of the null hypothesis of no-stationarity at the 5% level. Finally, the symbol *** denotes the rejection of the null hypothesis of no-stationarity at 1% level. For the KPSS test, the null hypothesis is that the series is stationary. In the lower part of the table, tests for unit roots as developed by Andrews and Zivot (1992) are performed for models A (shift level) and B (shift in rate of growth). For the nominal interest rate, we let the breakpoint be selected by the data using the Inf test on the ADF; its 5% critical level is –4.80 for model A and –4.42 for model B. For the FDM1 and ΔP series, we treat the breakpoint (λ) as given and the 5% critical level is –3.76 for model A and –3.96 for model B for λ=0.50. The chosen breakpoint at May of 1980 by the Inf ADF test represents λ=257/518=0.4961.

Table 2. OLS estimates of Basic and Modified Models (1959:1 to 2002:2):

Basic: $i_t = \delta_0 + \beta_1 (FDM1) + \varepsilon_t$

$i_t = \delta_0 + \delta_1 d(t) + \beta_1 (FDM1) + \varepsilon_t$

$i_t = \delta_0 + \delta_1 d(t) + \delta_2 TREND + \beta_1 (FDM1) + \varepsilon_t$

Modified: $i_t = \delta_0 + \beta_1 (FDM1) + \beta_2 (\Delta P) + \varepsilon_t$

$i_t = \delta_0 + \delta_1 d(t) + \beta_1 (FDM1) + \beta_2 (\Delta P) + \varepsilon_t$

$i_t = \delta_0 + \delta_1 d(t) + \delta_2 TREND + \beta_1 (FDM1) + \beta_2 (\Delta P) + \varepsilon_t$

	Basic Model		
Coefficients	**Without dummy**	**With dummy**	**With dummy and trend**
δ_0	6.641***	11.904***	12.602***
	(0.749)	(0.910)	(0.948)
δ_1		6.582***	5.130***
		(0.959)	(1.095)
δ_2			0.013***
			(0.004)
β_1	-0.365	-4.300***	-5.800***
	(0.262)	(0.594)	(0.672)
Adj. R^2	0.008	0.413	0.501

	Modified Model		
Coefficients	**Without dummy**	**With dummy**	**With dummy and trend**
δ_0	5.900 ***	10.368***	10.856***
	(0.475)	(0.773)	(0.788)
δ_1		5.386***	4.904***
		(0.757)	(0.821)
δ_2			0.006**
			(0.003)
β_1	-0.839***	-3.955***	-4.645***
	(0.210)	(0.490)	(0.538)
β_2	6.145***	4.808***	4.224***
	(0.910)	(0.835)	(0.805)
Adj. R^2	0.362	0.617	0.630

Notes: Monthly data are used as explained in the data section. The variables of the models are defined as follows: d(t) is the dummy variable defined as 0 if $t \leq [n\ \tau]$ and 1 if $t > [n\ \tau]$, where the unknown parameter $\tau \in (0,1)$ denotes the relative timing of the change point, t = 1, 2, …, n and [ ] denotes integer part, according to Gregory and Hansen (1996), the time trend (TREND) has value 1 in 1959:1 and grows monthly by one unit from then, **FDM1** stands for the federal debt to M1 ratio; **TB3** stands for the three months Treasury bill rate; and **ΔP** stands for the rate of inflation. ADF (k) refers to the Augmented Dickey-Fuller t-tests for cointegration as in Engle and Granger (1987), in which the null is there is no-cointegration. The lag length (k) is chosen by the Campbell-Perron data dependent procedure, as in Ng and Perron (1995). The method starts with an upper bound, k_{max}=18, on k. If the last included lag is significant, choose k = k_{max}. If not, reduce k by one until the last lag becomes significant (we use the 5% value of the asymptotic normal distribution to assess significance of the last lag). If no lags are significant, then set k = 0. Next to the ADF t-value, in parenthesis is the selected lag length. For the ADF tests, the symbol * attached to the figure indicates rejection of the null of no-stationarity at the 10% level. The symbol ** indicates rejection of the null hypothesis of no-stationarity at the 5% level. Finally, the symbol *** denotes the rejection of the null hypothesis of no-stationarity at 1% level.

Table 3. Cointegration Tests among Key Variables (1959:1-2002:2). Intercept (no trend) is included in cointegrating equation and test VAR.

Models ↓	Maximal Eigenvalue Test	5% Critical Value	Trace Test	5% Critical Value	VAR Lag Length [Criteria]	Null Hyp. on Coint. Vectors (C.V.)
BIVARIATE MODELS						
[FDM1, TB3]	6.79	14.07	8.57	15.41	7	**No C.V.s**
No Vector	1.79	3.76	1.79	3.76	[LR, FPE, AIC, HQ]	At most 1
[TB3, ΔP]						
Vector: TB3 ΔP	12.93	14.07	18.36*	15.41	8	**No C.V.s**
1 +19.32	5.43*	3.76	5.43*	3.76	[LR, FPE, AIC]	At most 1
(3.77)						
TRIVARIATE MODELS						
[TB3, FDM1, ΔP]						
Vector: TB3 FDM1 ΔP	43.55**	20.97	55.14**	29.68	5	**No C.V.s**
1 -1.90 +16.49	10.91	14.07	11.57	15.41	[LR, FPE, AIC]	At most 1
(0.44) (1.62)	0.68	3.76	0.68	3.76		
Including dummy 8005						
[TB3, FDM1, dum8005]						
Vector: TB3 FDM1 dum8005	21.98*	20.97	39.36**	29.68	7	**No C.V.s**
1 -9.91 +16.76	15.68*	14.07	17.38*	15.41	[LR, FPE, AIC, HQ]	At most 1
(1.84) (2.75)	1.70	3.76	1.70	3.76		At most 2
[TB3, ΔP, dum8005]					8	
Vector: TB3 ΔP dum8005	31.26**	20.97	45.60**	29.68	[LR, FPE, AIC]	**No C.V.s**
1 +35.94 -7.55	12.04	14.07	14.34	15.41		At most 1
(5.29) (1.98)	2.30	3.76	2.30	3.76		
[TB3, FDM1, ΔP, dum8005]						
Vector: TB3 FDM1 ΔP dum8005	52.55**	27.07	94.79**	47.21	5	**No C.V.s**
1 -3.37 +17.80 +2.16	23.77*	20.97	42.25**	29.68	[LR, FPE, AIC]	At most 1
(0.83) (1.67) (1.29)	17.57*	14.07	18.47*	15.41		
	0.91	3.76	0.91	3.76		

Notes: The variables are defined as in table 1. The data in levels have linear trends but cointegrating equation has only intercepts. The symbols ** indicates significance (rejection of the null hypothesis) at the 1% level and * indicates significance at the 5% level. Below the calculated values of the estimated coefficients by the Johansen cointegration method are marginal significance levels (p-values) with the exact probability. From the k-order VAR model, the ΔX_t and X_{t-k} are regressed on a constant and ΔX_{t-1}, …, ΔX_{t-k+1}. The obtained residuals R_{0t} and R_{kt} are used in the construction of the residual product matrices matrix S_{ij}. The matrix of cointegrating vectors is then estimated as the eigenvectors associated with the eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_r > 0$ found as the solution to $|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}|$. The test statistics (for n=2,3,4 variables in the system) are based on the maximal eigenvalue test and the trace test. The maximal eigenvalue test (null is r cointegration vectors against the alternative of r+1 cointegration vectors) is based on the statistic: $\lambda_{max} = -T\ln(1-\lambda_{r+1})$, where T is the sample size, r is the number of cointegrating vectors, and λ_i are the eingenvalues above. The trace test (null is at most r cointegration vectors against the alternative of more than r cointegration vectors) is based on the statistic:

$\lambda_{trace} = -T \sum_{i=r+1}^{p} \ln(1-\lambda_i)$. The lag-length selection criteria used were given by combination of the sequential modified likelihood ratio (LR) test, the final prediction error (FPE), and the Akaike (AIC), Schwarz (SIC) and Hannan-Quinn (HQIC) information criteria. Below the used lag-length for each specification are the criteria adopted.

Figure 1. U.S. series (1959:1-2002:1): TB3: Nominal 3-month interest rate; DP: inflation rate; FDM1: ratio of federal debt to M1; FDM2: ratio of federal debt to M2; and FDM3: ratio of federal debt to M3.

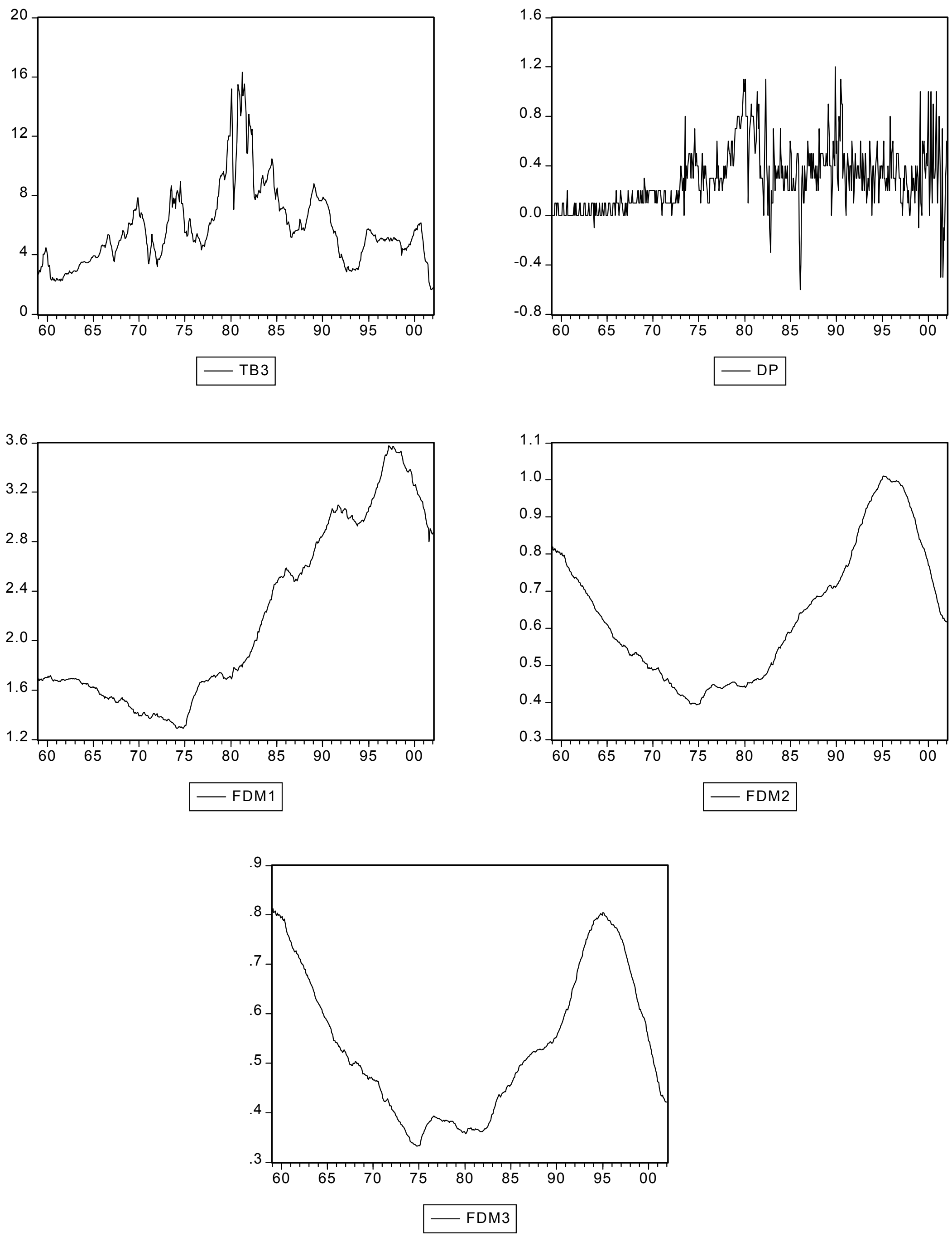
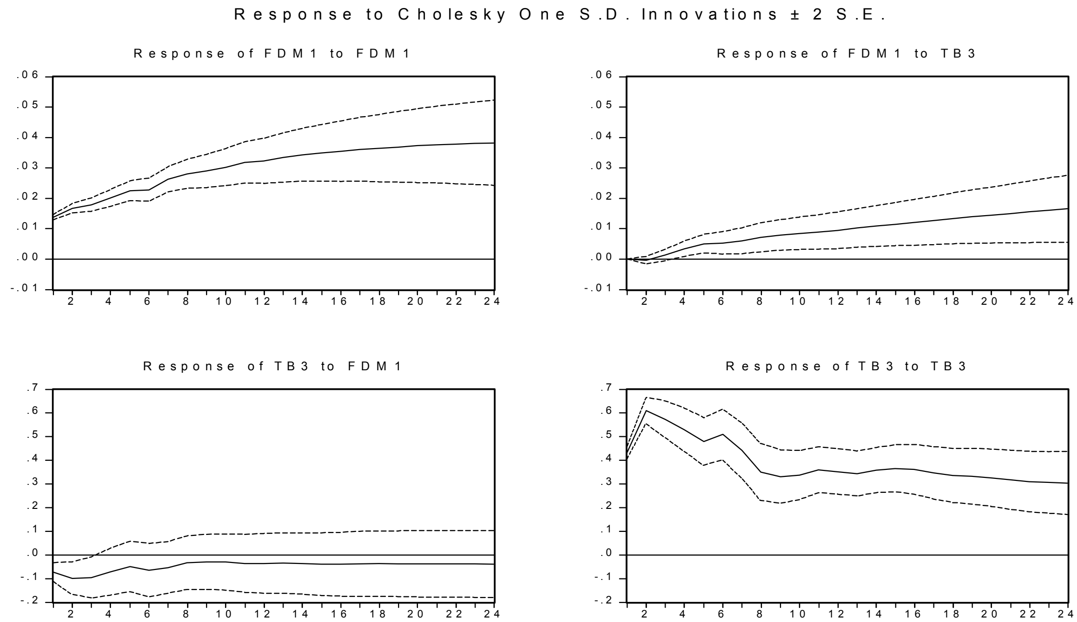


Figure 2. Impulse Responses of VAR [FDM1, TB3] and [Δ(FDM1), Δ(TB3)]



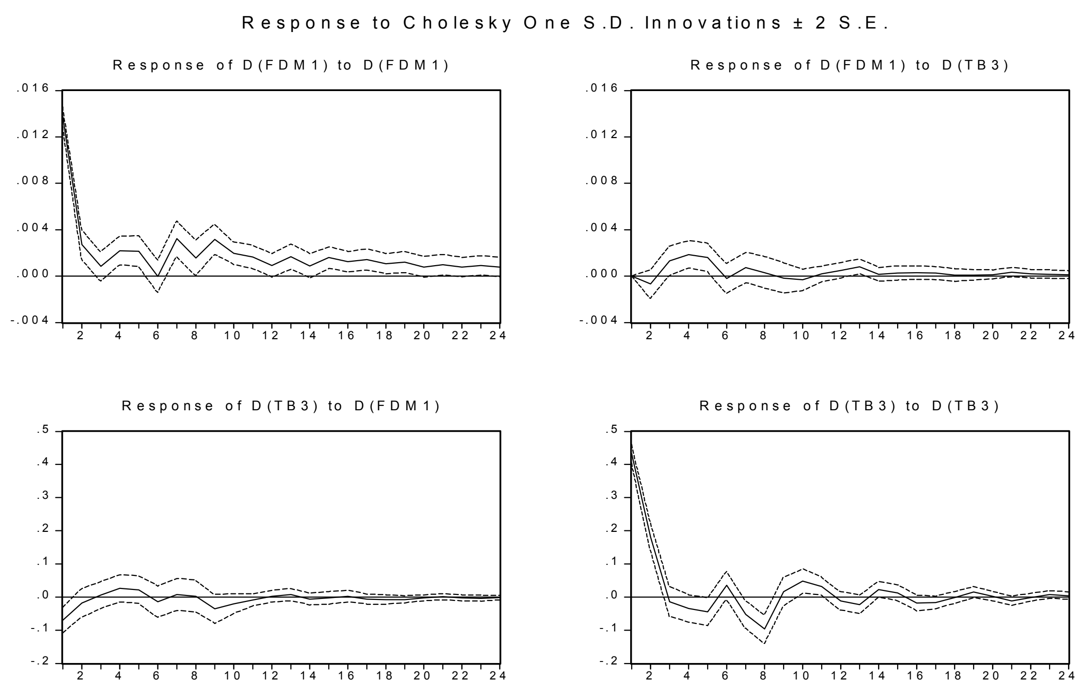


Figure 3. Impulse Responses of VAR [ΔP, FDM1, TB3] and [Δ(ΔP), Δ(FDM1), Δ(TB3)]