Common Nonlinearities in Long-Horizon Stock Returns: Evidence from the G-7 Stock Markets

Sei-Wan Kim[a], André V. Mollick[b], Kiseok Nam[c,*]

Department of Economics, Ewha Womans University, Seoul, Korea
Department of Economics & Finance, University of Texas Pan American, Edinburg, TX 78539, USA
Sy Syms School of Business, Yeshiva University, New York, NY 10033, USA

Abstract

Employing annual returns generated from overlapping monthly price indexes for the G-7 stock markets, this paper examines asymmetry and common nonlinearities in long-horizon stock returns. Identifying widespread nonlinearities based on LSTAR or ESTAR models, we find that the asymmetric nonlinear dynamics induces a substantial portion of predictable variations in long-horizon stock returns. The nonlinear models clearly outperform linear models "in sample" and in most of the out of sample forecasting exercises. With nonlinear impulse responses suggesting strong stability of return dynamics, the empirical results of this paper provide useful information in developing annual investment strategies for international stock markets.

Keywords: Long-horizon stock returns, Nonlinearities, Smooth Transition Autoregressive Model.

---

1. Introduction

A proper understanding of the stochastic processes followed by asset returns is central to the study of finance. Among the fundamental topics is the understanding of predictable time variations in both short and long-horizon stock returns. Investigating the data generating process

[*] Corresponding author. Tel.: +1 212 960 5400 Ext.5837; fax: +1 212 960 0824
E-mail address: knam@yu.edu (K. Nam).

in stock returns, recent research has suggested that the factors affecting the asset pricing behavior of investors are better described by nonlinear relationships with expected returns. Several recent studies have documented that predictable components of stock returns exhibit a strong nonlinearity induced from an asymmetric dynamic process. For instance, LeBaron (1992) and Koutmos (1998) explore an asymmetry in autoregressive processes of high frequency return series. Studies by Nam et al. (2002, 2003) identify nonlinearities induced from an asymmetric reverting property of weekly and monthly returns for U.S. and Pacific Basin equity markets. Skalin and Teräsvirta (1999), McMillan (2001), and Bradley and Jansen (2000, 2004) provide comparisons between linear and nonlinear models of short-horizon returns. [1]

The primary focus of these studies, however, is the identification of nonlinearities in short-horizon stock returns. In spite of its importance for understanding the equilibrium price behavior in the long-term, there have been few studies on nonlinearities in long-horizon stock returns. Previous works on long-horizon predictability have mostly dealt with the behavior of stock returns in a linear model context.

In an attempt to fill this gap, this paper examines the evidence on nonlinearities in long-horizon stock returns based upon the smooth transition autoregressive (STAR) models. We find widespread evidence of asymmetric dynamics in long-horizon as a common nonlinearity across G-7 major stock market indices. Nonlinear models clearly outperform linear models "in sample"

[1] Significant nonlinearities have been found in exchange rates as well. See Panos et al. (1997) and further reconsiderations in Sarantis (1999) and Taylor and Peel (2000), among others. Also, the particular form of non-linearity adopted varies across studies. For example, Chelley-Steeley (2005) employs nonlinear autoregressive models for several Eastern European stock markets. Massoumi and Racine (2002) develop a metric entropy capable of detecting nonlinear dependence within the returns series, while Kanas (2005) employs nonlinear nonparametric techniques to the stock price-dividend relation of major stock markets.

and in most of the out-of-sample forecasting exercises. Applying the non-linear impulse response function (NIRF) developed by Koop et al. (1996) and Potter (2000), our analysis also indicates that the STAR models in general lead to stabilized dynamics of long-horizon stock returns. Contrary to Sarantis (2001), we document strong long term stability features of U.S. and German stock markets from the impulse response functions. [2]

The empirical results of this paper provide valuable clues for a better understanding of the mean reversion property of stock prices, as well as rendering useful information in developing long-term investment strategies. The mean reversion property refers to the tendency of stock price to revert to its fundamental value. It has stood out as an important issue in the predictability of long-horizon stock returns. Summers (1986), Poterba and Summers (1988) and Fama and French (1988) have documented that stock prices take long temporary swings away from its fundamental value, and the transitory components cause a significant proportion of

[2] There are also several studies employing different time periods or methodologies to address the question of nonlinear predictability. Applying spectral tests that are robust to the presence of time-varying volatility, McPherson et al. (2005) find that Canada's TSE 100, Italy's BIC and the U.K.'s FTSE 100 (daily) index returns do show a degree of predictability. Studying nine macro variables as predictors of (monthly) stock returns, Rapach et al. (2005) find that interest rates are more consistent and reliable predictors of stock returns and that this holds for a large number of industrialized countries. International evidence on long memory in stock index data for eighteen countries is explored in Cheung and Lai (1995), who use a modified rescaled range (R/S) procedure to detect long memory. Richards (1997) shows that small markets are subject to larger reversals than large markets, maybe due to some form of market imperfection. Rouwenhorst (1998) documents return continuation in all twelve sample countries, lasting one year on average. Interestingly, Fama (1990) provides estimates of industrial production growth on contemporaneous and lagged NYSE returns and concludes that the regression *R*-square increases with the return horizon: from 0.06 for monthly returns to 0.43 for annual returns. Chiao et al. (2004) and Malliaropulos and Priestley (1999) discuss predictable components in South East Asian stock markets.

predictable variations in stock returns. Balvers et al. (2000) also find strong evidence of mean reversion in relative national stock index prices.

The nonlinear dynamics explored in this paper indicates that long-horizon stock prices exhibit an asymmetric mean reverting behavior. Depending on the level of a specific lagged return (as a transition variable), the speed of mean reversion is uneven under different states of return persistence. The empirical results for the U.S. market, for example, indicate that the annual return dynamics exhibits persistence under a positive prior return, while showing a strong mean reversion behavior under a negative prior return.

The practical implication of the asymmetric mean reversion lies in its usefulness in developing the long-term portfolio strategies, such as the contrarian and momentum portfolio strategies or the negative and positive feedback strategy for individual stocks and indexes. [3] The contrarian strategy was introduced by DeBondt and Thaler (1985) who showed that a simple trading strategy of buying "recent losers" and selling "recent winners" yields a substantial excess profit in long-term investment horizons. In contrast, Jegadeesh and Titman (1993) show that a momentum trading strategy of buying "recent winners" and selling "recent losers" can yield an abnormal profit. While a momentum trading exploits price momentum (continuation), the contrarian profits result from exploiting the mean-reverting behavior of stock price. In the same nature of trading, negative (positive) feedback traders buy an individual stock or index after the price falls (rises).

[3] The terms "contrarian" and "momentum" are typically used for the portfolio trading rules. The corresponding terms for a single stock or an index trading are "positive feedback" and "negative feedback" trading. Positive feedback traders buy an individual stock or stock index when its price rises, while negative feedback traders buy a stock or index when its price falls. These terms are thus used somewhat interchangeably.

Several implications for investment strategies are possible from our empirical results. For a prior negative return, all indexes except for Japan show a strong mean reverting tendency, while Japan, U.K. and U.S. indexes exhibit return persistence under a prior positive return. The implications are as follows: (a) a positive feedback trading on the index for Japan could yield an abnormal profit due to its significant return persistence; (b) given its significant return persistence, the Japanese stock market might be in favor of the momentum portfolio strategy in annual investment horizon; (c) all the indexes except for Japan could be individually used for a negative feedback trading; and (d) given their strong mean reverting pattern of a prior negative return, all the markets except for Japan might be favorable to the contrarian portfolio strategy for 2-3 years of investment horizon.

The remainder of the paper is organized as follows. In section 2, we discuss the data construction. Section 3 introduces the estimated non-linear models, and the empirical results are presented in section 4. Section 5 concludes the paper.

2. Annualized Returns from Overlaps of Monthly Prices

One of the difficulties in analyzing long-horizon stock returns is the insufficient numbers of observations available. In general, the number of non-overlapping observations of annual returns for most of national stock markets is too small to be used for reliable statistical analysis. In this paper, we handle this problem by generating enough annual returns from the overlap of monthly prices. For each month, we generate annual stock price growth rates from the logarithmic difference of monthly price index over the same month in the previous year.

Fama and French (1988) employ this methodology to obtain enough observations of annual returns to examine negative first-order return autocorrelation. However, while Fama and French (1988) estimate first-order autocorrelation of long-horizon returns using the linear AR (1) model, we estimate higher order autocorrelation in the nonlinear AR (*p*) model. Sarantis (2001) also adopt the same methodology to generate annualized return series. However, there are two major differences between our analysis and theirs. First, while Fama and French (1988) estimate only first-order autocorrelation of long-horizon returns using the linear AR (1) model, we estimate higher order autocorrelation in the *nonlinear* AR (*p*) model. Second, the way we compute the lagged annual returns for lagged variables is completely different from the approach in Sarantis (2001).

Consider the U.S. stock market for illustration purposes. Data on the S&P 500 index run from the last day of January of 1965 to the last business day of September of 1999. We obtain the January 1966 growth rate of the stock index as the logarithmic of prices between January 1966 and January 1965. This is done month by month until the end-date. We call this series the contemporaneous annualized returns. It runs from January 1966 until September 1999, exactly as in Sarantis (2001). Suppose the current annual return on January 1974 is defined as $r_t = \ln(P_{Jan74} / P_{Jan73})$. Then r_{t-1} (the annualized return on January 1973) is computed as $r_{t-1} = \ln(P_{Jan73} / P_{Jan72})$, r_{t-2} is computed as $r_{t-2} = \ln(P_{Jan72} / P_{Jan71})$, and successively until r_{t-8} as the annual return on January 1966 is computed as $r_{t-8} = \ln(P_{Jan66} / P_{Jan65})$. We do the same for annual return on February 1974 as $r_t = \ln(P_{Feb74} / P_{Feb73})$, lagging the return series r_{t-1}, r_{t-2}, …, r_{t-8}, and so on until the return series of September 1999. In contrast, in Sarantis (2001), lagged returns are defined differently: for the annual return on January 1974, i.e., $r_t = \ln(P_{Jan74} / P_{Jan73})$,

its lagged returns are computed as $r_{t-1} = \ln(P_{Dec73} / P_{Dec72})$, $r_{t-2} = \ln(P_{Nov73} / P_{Nov72})$, and so on. In other words, the lagged returns in Sarantis (2001) are obtained from following the conventional way of treating non-overlapping time series observations. It turns out that the lagging period is not annual-based but monthly based in his case. Therefore, although employing the same annualized returns for estimation, the autoregressive models specified in Sarantis (2001) do not describe the dynamics of annual returns but rather describe the dynamics of annualized returns in monthly frequency.

The above procedure generates a sequence of observations that will be used when estimating the linear and nonlinear models. Generating annual stock returns in a different fashion from previous works, we list three implications: first, the number of observations increases substantially, thereby increasing statistical power; second, our method of treating lagged variables provides a new dimension to time-series analysis of long-horizon stock returns; and third, our methodology makes it possible to compare our results (for long-term dynamics) with Sarantis's (for short-term dynamics).

3. The Model

Focusing on modeling stock returns as a non-linear and state-dependent, we employ in this paper the smooth transition autoregressive (STAR) models to describe the smooth transition of return dynamics in long-horizon. [4] The main feature of the STAR model is the switching-

[4] There are two types of switching models: the threshold autoregressive (TAR) model developed by Tsay (1989) and the smooth transition autoregressive (STAR) model developed by Luukkonen, Saikkonen, and Teräsvirta (1988), Teräsvirta and Anderson (1992), and Teräsvirta (1994). While the TAR model specifies a sudden transition

regime mechanism to allow dynamics of stock returns evolving with a smooth transition between regimes that depends on the sign and magnitude of past realization of returns.

For stock returns r_t, we specify the following STAR model of order p to capture the nonlinearities characterized by asymmetries in return dynamics:

$$\begin{aligned} r_t &= [\phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i}] + [\rho_0 + \sum_{i=1}^{p} \rho_i r_{t-i}] \cdot F(r_{t-d}) + \varepsilon_t \\ &= [\phi_0 + \phi(L) r_t] + [\rho_0 + \rho(L) r_t] \cdot F(r_{t-d}) + \varepsilon_t , \end{aligned} \tag{1}$$

where $F(\cdot)$ is the transition function that controls the regime-shift mechanism. It is a smooth and continuous function of past realized returns. In our STAR model, nonlinearities arise through conditioning on lagged stock returns, such that past realized return r_{t-d} is the transition variable. d is the delay parameter showing the number of periods that the transition variable leads the switch in dynamics.

STAR model comes in two types: the logistic smooth transition autoregressive (LSTAR) model and the exponential smooth transition autoregressive (ESTAR) model. The LSTAR model better describes a stochastic process that is characterized by an alternative set of dynamics for either large or small value of the transition function. In the LSTAR model the transition function is given by the following logistic function:

$$F(r_{t-d}) = [1 + \exp\{-\gamma(r_{t-d} - c)\}]^{-1}, \quad \gamma > 0 \tag{2}$$

---

between regimes with a discrete jump, the STAR model allows a smooth transition between regimes in the dynamics. See Teräsvirta et al. (2004) for a survey of the literature.

The logistic function, $F(r_{t-d})$, takes a value between 0 and 1 depending on the degree and direction by which r_{t-d} deviates from *c*, the switching value of the transition variable. The estimated value for *c* defines the rough line of transition between two regimes, such that $0 < F(r_{t-d}) < 0.5$ (a lower regime) for $r_{t-d} < c$ and $0.5 < F(r_{t-d}) < 1$ (a upper regime) for $r_{t-d} > c$. When $r_{t-d} = c$, $F(r_{t-d}) = 0.5$ such that that current dynamics of r_t is half way between the upper and lower regimes. Specifically, when r_{t-d} takes an extremely large value, i.e., $r_{t-d} >> c$, $\exp\{-\gamma(r_{t-d} - c)\}$ is close to zero. As a result, the value of $F(r_{t-d})$ approaches one and the dynamics of r_t is generated by both the ϕ_i and ρ_i parameters in equation (1). In addition, for the extremely small value of r_{t-d}, i.e., $r_{t-1} << c$, $\exp\{-\gamma(r_{t-d} - c)\}$ is close to one. Then, the value of the transition function $F(r_{t-d})$ approaches one and the dynamics of r_t is generated by the ϕ_i only. [5]

In contrast, the ESTAR model is more appropriate to generate alternative dynamics for both large and small value for the transition variable. In the ESTAR model, the transition function is given by:

$$F(r_{t-d}) = 1 - \exp\{-\gamma(r_{t-d} - c)^2\}, \tag{3}$$

[5] There is no specific relation between the sign of *c* and the trend of stock returns. The parameter *c* simply represents the threshold value of the transition variable to trigger the regime-switching. As the transition variable takes a value above or below the threshold, return dynamics evolve in a different autoregressive process, thereby generating an asymmetry in the long swings. Thus, depending on the asymmetry nature of each individual return dynamics, the estimated value of *c* can be positive or negative.

where for the extremely large or small value of r_{t-d}, the value of $\exp\{-\gamma(r_{t-d}-c)^2\}$ approaches zero and the value of the transition function approaches one. The dynamics of r_t is then generated by both the ϕ_i and ρ_i parameters in equation (1). When the value of r_{t-d} is close to *c*, the value of $\exp\{-\gamma(r_{t-d}-c)^2\}$ approaches one and the value of the transition function approaches zero. In these cases, the dynamics of r_t is generated by the ϕ_i parameters only.

The adjustment parameter γ in both models governs the speed of transition between the two regimes, such that the greater the value of γ the faster the transition between the regimes. In the limit, as the value of γ approaches infinite, the model degenerates into the conventional threshold autoregressive (TAR) model by Tsay (1989). Alternatively, if γ approaches zero such that the value of the transition function $F(r_{t-d})$ approaches one, then the model degenerates to the linear AR model, with the ϕ_i and ρ_i parameters unidentifiable. For the choice between LSTAR and ESTAR, we follow the procedure suggested by Terasvirta and Anderson (1992), which is described in the next section.

4. Empirical Applications

4.1. Data Description

We employ monthly stock market index data for the G-7 OECD countries: Canada (Toronto 300 Composite), France (DS market), Germany (Dax 100), Italy (Milan Bourse), Japan (Tokyo New Stock Exchange), the U.K. (FTSE All Shares), and the U.S. (S&P 500). While the starting date varies for each country, the ending date of observations is September 1999 for all

countries. The data peculiarities make the U.S. series display the longest time period (1974:01-1999:09) together with the U.K. Then Canada and Japan follow suit with time periods running from 1978:01 to1999:09, while France, Germany and Italy share the 1982:01 to 1999:09 time period. The data are retrieved from *Datastream* and the sample period is selected to make our results comparable to Sarantis (2001), who also used the annual stock price growth rate as the logarithmic difference over the same month in the previous year. See Section 2.

Summary statistics for annualized index returns of each country are presented in Table 1. There are no significant variations in mean and standard deviation among all seven index returns. Except for Italy and Japan, which show positive skewness, five other indexes show negative skewness. Interestingly, however, only the U.K. exhibits excess kurtosis (or fatter tail). This indicates that long-horizon index returns may not be associated with volatility clustering, which is commonly observed in short-horizon stock returns. Except for Germany and Japan, however, all other indexes still exhibit non-normality features.

4.2. The Identification of STAR models

First, we identify and estimate a benchmark linear AR model. We select the appropriate lag length k on the basis of the Akaike Information Criterion (AIC) over a range of lags from 1 through 8. The selected lag length for each country is reported in Table 2.

We next perform the linearity test to examine whether the linear AR model is appropriate against a nonlinear STAR alternative. Following Teräsvirta and Anderson (1992), we estimate the following auxiliary regression:

$$r_t = \phi_0 + \sum_{i=1}^{k} \phi_{1,i} \cdot r_{t-i} + \sum_{i=1}^{k} \phi_{2,i} \cdot r_{t-i} r_{t-d} + \sum_{i=1}^{k} \phi_{3,i} \cdot r_{t-i} r_{t-d}^2 + + \sum_{i=1}^{k} \phi_{4,i} \cdot r_{t-i} r_{t-d}^3 + \varepsilon_t , \tag{4}$$

where the linearity test becomes $H_{01} : \phi_{2i} = \phi_{3i} = \phi_{4i} = 0$, for all i. Table 2 contains the results for the linearity tests, with the maximum lag length for k. Four of the stock markets have an autoregressive lag of 8, while France and the U.S. have shorter lag-lengths at 4 and 5, respectively. Reported in the table is the maximum F-statistics for each delay lag, in which the delay parameter varies over the range $1 \le d \le 4$. The estimate of the delay parameter d is chosen by the highest F-statistics, which is marked with an asterisk in Table 2. The value of the delay parameter varies between 1 and 2 across stock markets, while Canada has $d = 3$ and France has $d = 4$. From Table 2, it is clear the rejection of linearity across all stock markets.

Given that linearity is rejected for all the seven countries, we next specify an appropriate STAR model to capture nonlinear dynamics of stock returns for each country. As suggested by Teräsvirta and Anderson (1992), the linearity test can be used to provide a sequence of nested hypothesis tests H_{04}, H_{03}, H_{02} for the choice between LSTAR and ESTAR alternatives. The sequence of nested tests for the coefficients in equation (4) above implies:

$$\begin{aligned} &H_{04} : \phi_{4i} = 0 , && i = 1, \cdots, k \\ &H_{03} : \phi_{3i} = 0 / \phi_{4i} = 0 , && i = 1, \cdots, k \\ &H_{02} : \phi_{2i} = 0 / \phi_{3i} = \phi_{4i} = 0 , && i = 1, \cdots, k \end{aligned} \tag{5}$$

Rejection of H_{04} implies selecting the LSTAR model. If H_{04} is not rejected and H_{03} is rejected, the ESTAR model is chosen. Not rejecting H_{04} and H_{03} and rejecting H_{02} leads to an

LSTAR model. It should be noted that if none of the null hypothesis can be rejected, then the linearity cannot be rejected, such that the linear AR model should be selected. Table 3 contains the full results on these tests. Reported are the *p*-values associated with each hypothesis test. One is able to reject at a 5% significance level for all stock markets but two, which suggests LSTAR models in general. For Germany and the U.K. it is possible to infer that H_{04} is not rejected and H_{03} is rejected, which leads to the ESTAR model.

4.3. Estimation results and interpretations

We first estimate the linear benchmark AR model for each series. The maximum lag length for each return series is selected by the AIC statistic. Table 4 reports the estimation results. The interesting finding is that all indexes, except Japan, exhibit negative first-order autocorrelation. It implies that annual stock returns are strongly negatively autocorrelated. This result is consistent with the mean reversion property of long-horizon stock prices explored by Summers (1986) and Fama and French (1988). Table 4 also reports R^2 and mean-squared error of regression for each series to compare with those from the nonlinear STAR models.

Table 5 reports the estimation results of LSTAR and ESTAR models conducted under the nonlinear least-squares (NLS) method. Following Teräsvirta (1994), we standardize the exponent of the transition function $F(\cdot)$ to make the γ-parameter scale-free and divide the exponent of F by the standard deviation (σ) of return series r_t for the LSTAR model and by the variance (σ^2) for the ESTAR model.

Several observations are worth mentioning in Table 5. First, the value of γ-parameter is always positive and statistically significant at the 5% level for all countries, except for Italy. This

indicates that transition between regimes of return dynamics is significant for most of the markets. Second, the c-parameter indicates the halfway point between the expansion and contraction phases of the equity markets. In all cases, except for Germany's ESTAR model, the estimated value of c is statistically significant at the 5% level. The estimated value of c is positive for Canada, France, the U.K, and the U.S., while it is negative for Germany, Italy, and Japan. This implies that for each market a different level of return shock triggers regime shifts. Third, most of the ρ-parameters are statistically significant at the 5% level for all series. In general, the STAR models yield a relatively high level of R^2, which indicates that substantial portion of time variations in long-horizon stock returns is associated with nonlinear dynamics. Also, in comparison with the results of the benchmark linear AR models, those of nonlinear STAR models show dramatic improvements in R^2 and standard error of regression (SER) for all indexes. For example, the nonlinear model for Germany yields R^2 of 63.2% and SER of 0.127, while the linear model yields R^2 of 32.2% and SER of 0.169.

In sum, the empirical results strongly indicate that the STAR models well capture the nonlinear dynamics induced by an asymmetry in mean reverting behavior of each series for all markets. While the asymmetric reverting pattern varies for each series, a stronger reverting pattern of a negative return is commonly observed for all markets. This implies that a negative return on average exhibits a greater speed of reversion. However, only Japan, the U.K. and the U.S. indexes exhibit return persistence under a prior positive return, which implies that an increase in stock price is expected to persist for a while. Interestingly, Canada, France, Germany, and Italy exhibit a negative autocorrelation even under a prior positive return. In particular, results for Canada and France show that negative autocorrelation under a prior positive return is much greater than that under a prior positive return. This implies that annual stock index for

Canada and France has a mean-reverting behavior to its fundamental value after a significant price increase of more than 24.1% and 29.2% (the value of *c* in Table 5 is 0.241 for Canada and 0.292 for France). However, the index for Japan exhibits a positive autocorrelation (or return persistence) under both a prior positive and a negative return.

The empirical results of this paper provide useful information for developing annual investment strategies of international stock markets in terms of positive or negative feedback trading strategies for each individual index or international portfolios. According to the results, the following strategies are possible: (a) a positive feedback trading on the index for Japan could yield an abnormal profit due to its significant return persistence; (b) given its significant return persistence, the Japanese stock market might be in favor of the momentum portfolio strategy in annual investment horizon; (c) all the indexes except for Japan could be individually used for a negative feedback trading; and (d) given their strong mean reverting pattern of a prior negative return, all the markets except for Japan might be favorable to the contrarian portfolio strategy for 2-3 years of investment horizon.

4.4. The Dynamic behavior

We investigate the dynamic behavior of the STAR models by examining the characteristic roots of the models derived from estimations. Characteristic roots are computed from the following characteristic polynomial:

$$\lambda^k - \sum_{j=1}^{k} (\phi_j^k + \rho_j^k F)\lambda^{k-j} = 0, \tag{6}$$

where $\lambda's$ are characteristic roots for k^{th} order, ϕ_j^k is a vector of $(\phi_1, \cdots, \phi_k)$, and ρ_j^k is a vector of $(\rho_1, \cdots, \rho_k)$. First, we calculate the roots for the regime with $F = 0$, which corresponds to the lower (or contraction) regime in the LSTAR model and the middle regime in the ESTAR model. We next calculate the roots for the regime with $F = 1$, which describes the upper (or expansion) regime in the LSTAR model and the outer (either expansion or contraction) regime in the ESTAR model.

Table 6 presents the characteristic roots for each regime for all countries. Both regimes include pairs of complex roots for most of the countries. This suggests that stock markets are characterized by cyclical movements during both the expansion and contraction phases, and the STAR models well describe asymmetric behavior of long-horizon stock returns for all countries. Except for the U.S., each country shows that at least one regime includes one or more explosive roots. While the middle regime for Germany is dominated by an explosive root, the middle regime for the U.K. is stable. This indicates that, while German stock returns pass through the middle regime very quickly on the way up or down, U.K. stock returns tend to stay in the middle regime. Outer regime for both countries includes explosive roots, which leads to a quicker transition of return dynamics to the middle regime. In the case of Canada, Italy, and Japan, both the upper and lower regimes include explosive roots, thereby indicating that the stock market appears to be less stable in both regimes. For France, the lower regime is stable and only the upper regime includes explosive roots. Thus, once stock returns are in the lower regime they are more likely to stay there for a while, whereas returns in the upper regime tend to pass quickly. The following implication follows. While the recession of the stock return from a peak is always very rapid, once the stock market is in a contraction phase it tends to remain there. Interestingly, for the U.S. both regimes are stable.

4.5. The Out-of-sample forecasts

The forecasting performance of the STAR model is examined with the multi-step ahead out-of-sample forecasts and is compared to forecasting performance of the linear model in terms of root mean square prediction errors (RMSPE) and mean absolute prediction error (MAPE). We consider one, two, and three step ahead forecasts. We compute the ratio of RMSPE (MAPE) of the STAR model to the RMSPE (MAPE) of the benchmark linear AR model to measure relative forecasting performance between the nonlinear and linear models. A ratio of less than one indicates the STAR model outperforms the linear model, providing more accurate predictions in the out-of-sample forecast. Similarly, a ratio greater than one indicates the STAR model provides less accurate predictions than the benchmark linear model.

We follow Sarantis's (2001) rolling estimation procedure which updates the parameter estimates for an additional observation included in estimation. First, we re-estimate all models up to 1996:12 and we employ these estimates to generate multi-step ahead forecasts for the period 1997:1 – 1999:9. Second, we add observation of 1997:1 to the sample, re-estimate the models, and generate multi-step ahead forecasts for the period 1997:2 – 1999:9. We repeat this process of recursive model estimation and calculation of multi-step ahead out-of-sample forecasts.

Table 7 reports the ratios based upon the RMSPE and MAPE criteria for all three steps ahead forecasts. It shows that for the one- and two-step ahead forecasts, the STAR model outperforms the benchmark linear AR model for five (US, Japan, Germany, UK, and Italy) out of seven countries. For the three-step ahead forecast, the STAR model outperforms the benchmark

linear AR model for four (U.S., Germany, U.K., and Italy) out of seven countries. The results indicate that the STAR model in general can improve on the linear AR model.

4.6. Nonlinear impulse response functions

We next perform nonlinear impulse response analysis to examine dynamic stability of the STAR models. We calculate nonlinear impulse response functions to illustrate some of the dynamics of long-horizon stock returns in responding to the 1987 shock. As described in Potter (2000), where a conditional moment profile of a series is compared to some baseline profile, nonlinear impulse response functions in the regime-switching framework are state-dependent, asymmetric, and time-varying with the sign and size of the shock. Following Potter's (2000) definition, nonlinear impulse response functions for the stock returns r_t can be calculated as the difference between two conditional expectations:

$$NIRF_n(\delta, r_t, r_{t-1}, \cdots) = E[r_{t+n} \mid r_t = \hat{r}_t + \delta, r_{t-1} = \hat{r}_{t-1}, \cdots] - E[r_{t+n} \mid r_t = \hat{r}_t, r_{t-1} = \hat{r}_{t-1}, \cdots], \qquad (7)$$

where δ is the impulse and $\hat{r}$'s are realized values of the stock returns. The nonlinear impulse response functions are calculated as follows. First, we reserve the residuals for the sample period from the nonlinear estimation. Then, using the residuals, we get the baseline forecast and the perturbed forecast. To get the baseline forecast, for a shock at time t we randomly draw from reserved residuals and calculate the forecast from time $t+1$ to $t+37$. We average the realizations over 5,000 iterations to get the baseline path. To get the perturbed forecast, we repeat the exercise imposing the postulated shock δ at time t, with one- to three-standard-error

magnitudes. We again draw from the reserved residuals to calculate the forecast for periods $t+1$ through $t+37$, iterating 5,000 times and averaging.

Figure 1 presents impulse response functions for all countries for the 1987 shock. For all countries except Canada, the impulse response functions are stable and fairly symmetric for the same magnitude of positive and negative shocks. For Canada, however, the STAR model exhibits instability in return dynamics. As already mentioned in section 4.4, outer regime of return dynamics for Canada includes explosive roots, thereby leading to an unstable model dynamics. The impulse response functions for Japan exhibit an asymmetry, which is due to an explosive root dominated in the upper regime. Also, there are minor asymmetries in impulse responses for France and the U.K. Interestingly, impulse response functions for Germany and the U.S. show no significant responses to shocks, implying a strong stability for the STAR models. In sum, the impulse response function analysis indicates that the STAR models in general lead to stabilized dynamics of long-horizon stock returns.

5. Concluding Remarks

Employing *annualized returns for G-7 stock markets*, this paper examines the evidence on nonlinearities in long-horizon returns based on STAR models. We find widespread evidence of nonlinearity, which explains a substantial portion of time variations in the dynamics of long-horizon stock returns. Nonlinear models clearly outperform linear models "in-sample" and in most of the out-of-sample forecasting exercises. The nonlinear impulse response analysis suggests that the nonlinear models exhibit a strong stability of return dynamics. Nonlinear impulse response functions for German and U.S. markets show no significant responses to

shocks, implying strong stability of their corresponding STAR models, which is also found in general, except for Canada. Contrary to Sarantis (2001) who focuses on short-term dynamics, we document strong stability features of U.S. and German stock markets in longer term dynamics. Our STAR models outperform "in-sample" autoregressive linear models and the transition parameter on the regime shift is found to be statistically significant at all markets except Italy. While lending support to the notion that the non-linear dynamics of G-7 long-horizon stock returns is more profound than previously reported, the empirical results of this paper render useful information in developing long-term investment strategies for international stock markets.

Our results invite further examination as well. An extension to other sorts of nonlinear models is also possible. Recent research by Pérez-Rodríguez et al. (2005), for example, has shown that artificial neural networks (ANNs) provide better forecasts than the linear AR model and the STAR models for the Spanish stock market. The forecasts may also improve with interest rates as explanatory variables as recently documented by Rapach at al. (2005) for all stock markets. See also Estrella and Mishkin (1998) for the yield curve. These extensions are left for further research.

Acknowledgments

We wish to thank participants at the 2007 annual meeting of the Financial Management Association for their comments on this manuscript. We also thank the editor (Manuchehr Shahrokhi) and the two anonymous referees for their insightful comments and suggestions.

References

Balvers, R., Y. Wu, and E. Gilliland, 2000, Mean Reversion across National Stock Markets and Parametric Contrarian Investment Strategies, *Journal of Finance* 55 (2), 745-772.

Bradley, M. and D. Jansen, 2004, Forecasting with a Nonlinear Dynamic Model of Stock Returns and Industrial Production, *International Journal of Forecasting* 20*,* 321 - 342.

Bradley, M. and D. Jansen, 2000, Are Business Cycle Dynamics the Same across Countries? Testing Linearity around the Globe, *Studies in Nonlinear Dynamics & Econometrics* 4 (2), 51-71.

Chelley-Steeley, P., 2005, Modeling Equity Market Integration using Smooth Transition Analysis: A Study of Eastern European Stock Markets, *Journal of International Money and Finance* 24 (5), 818-831.

Cheung, Y. and K. Lai, 1995, A Search for Long Memory in International Stock Market Returns, *Journal of International Money and Finance* 14 (4), 597-615.

Chiao, C., K. Hung, and C. Lee, 2004, The Price Adjustment and Lead-Lag Relations between Stock Returns: Microstructure Evidence from the Taiwan Stock Market, *Journal of Empirical Finance* 11, 709-731.

DeBondt, W. F. M. and R. H. Thaler, 1985, Does the stock market overreact?, *Journal of Finance* 40, 793-805.

Estrella, A. and Mishkin, F., 1998, Predicting U.S. Recessions: Financial Variables as Leading Indicators, *Review of Economics and Statistics* 80, 45-61.

Fama, E. and K. French, 1988, Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96 (2), 246-273.

Fama, E., 1990, Stock Returns, Expected Returns, and Real Activity, *Journal of Finance* 45, 1089-1108.

Jegadeesh, N., and S. Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65-91.

Kanas, A., 2005, Nonlinearity in the Stock Price-Dividend Relation, *Journal of International Money and Finance* 24, 583-606.

Koop, G., Pesaran, M. H., and Potter, S., 1996, Impulse Response Analysis in Nonlinear Multivariate Models, *Journal of Econometrics* 74*,* 119-47.

Koutmos, G., 1998, Asymmetries in the Conditional Mean and the Conditional Variance: Evidence from Nine Stock Markets, *Journal of Economics and Business* 50*,* 277-290.

LeBaron, B., 1992, Some Relations between Volatility and Serial Correlations in Stock Market Returns, *Journal of Business* 65*,* 199-219.

Luukkonen, R., Saikkonen, P., and Teräsvirta, T., 1988, Testing Linearity against Smooth Transition Autoregressive Models, *Biometrika 75*, 491-499.

Maasoumi, E. and J. Racine, 2002, Entropy and Predictability of Stock Market Returns, *Journal of Econometrics* 107, 291-312.

Malliaropulos, A. and R. Priestley, 1999, Mean Reversion in Southeast Asian Stock Market, *Journal of Empirical Finance* 6, 355-384.

McMillan, D., 2001, Nonlinear Predictability of Stock Market Returns: Evidence from Nonparametric and Threshold Models, *International Review of Economics and Finance* 10, 353-368.

McPherson, M., J. Palardy, and J. Vilasuso, 2005, Are International Stock Returns Predictable? An Application of Spectral Shape Tests Corrected for Heteroskedasticity, *Journal of Economics & Business* 57, 103-118.

Nam, K., C. S. Pyun, A. C. Arize, 2002, Asymmetric Return Reversals and Contrarian Profits: ANST-GARCH Approach, *Journal of Empirical Finance* 9, 475-608.

Nam, K., C. S. Pyun, and S. W. Kim, 2003, Is Asymmetric Mean-Reverting Pattern in Stock Returns Systematic? Evidence from Pacific-Basin Markets in the Short-Horizon, *Journal of International Financial Markets, Institutions & Money* 13, 481-502.

Panos, M., A. R. Nobay, and D. Peel, 1997, Transactions Costs and Nonlinear Adjustment in Real Exchange Rates: An Empirical Investigation", *Journal of Political Economy* 105 (4), 862 - 879.

Pérez-Rodríguez, J., S. Torra, and J. Andrada-Félix, 2005, STAR and ANN Models: Forecasting Performance on the Spanish "Ibex-35" Stock Index, *Journal of Empirical Finance* 12, 490-509.

Poterba, J. and L. Summers, 1988, Mean Reversion in Stock Prices, *Journal of Financial Economics* 22, 27-59.

Potter, S., 2000, Nonlinear Impulse Response Functions, *Journal of Economic Dynamics and Control* 24, 1425-1446.

Rapach, D., M. Wohar, and J. Rangvid, 2005, Macro Variables and International Stock Return Predictability, *International Journal of Forecasting* 21*,* 137 - 166.

Richards, A., 1997, Winner-Loser Reversals in National Stock Market Indices: Can they be Explained?, *Journal of Finance* 52 (5), 2129-2144.

Rouwenhorst, K. G., 1998, International Momentum Strategies, *Journal of Finance* 53, 267-284.

Sarantis, N., 2001, Nonlinearities, Cyclical Behaviour and Predictability in Stock Markets: International Evidence, *International Journal of Forecasting* 17*,* 459 - 482.

Sarantis, N., 1999, Modeling Non-linearities in Real Effective Exchange Rates, *Journal of International Money and Finance* 18*,* 27 - 45.

Skalin, J. and Teräsvirta, T. (1999), Another Look at Swedish Business Cycles: 1861 – 1988, *Journal of Applied Econometrics* 14*,* 359 - 378.

Summers, L., 1986, Does the Stock Market Rationally Reflect Fundamental Values? *Journal of Finance* 41*,* 591 - 601.

Taylor, M. and Peel, D., 2000, Nonlinear adjustment, long-run equilibrium and exchange rate fundamentals, *Journal of International Money and Finance* 19, 33-53.

Teräsvirta, T., 1994, Specification, estimation and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association* 89, 208-218.

Teräsvirta, T. and Anderson, H. M., 1992, Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models, *Journal of Applied Econometrics* 7, S119-S136.

Teräsvirta, T., D. van Dijk, and M. Medeiros, 2004, Linear Models, Smooth Transition Autoregressions, and Neural Networks for Forecasting Macroeconomic Times Series: A Re-examination, *Stockholm School of Economics*, Working Paper Series No. 561, October.

Tsay, R., 1989, Testing and Modeling Threshold Autoregressive Processes, *Journal of the American Statistical Association 84*, 231-40.

Table 1
Summary Statistics of Stock Returns (r_t).

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Period	78:01 – 99:09	78:01 – 99:09	82:01 – 99:09	81:01 – 99:09	78:01 – 99:09	74:01 – 99:09	71:01 – 99:09
Mean	0.071	0.139	0.111	0.134	0.094	0.090	0.095
Std. Dev	0.184	0.207	0.201	0.320	0.272	0.232	0.147
Maximum	0.626	0.590	0.574	1.073	0.849	0.754	0.428
Minimum	-0.617	-0.447	-0.446	-0.488	-0.533	-0.905	-0.534
Skewness	-0.331	-0.348	-0.154	0.659	0.237	-1.175	-0.717
Kurtosis	3.820	2.714	2.675	3.224	2.702	7.234	3.917
JB-Test	12.07 (0.00)	6.16 (0.05)	1.77 (0.41)	16.76 (0.00)	3.41 (0.18)	301.87 (0.00)	41.65 (0.00)

Notes: Skewness and Kurtosis are presented to examine whether the third and fourth moments of r_t conform to a normally distributed random variable. The JB-Test refers to the Jarque-Bera normality test, in which the null hypothesis is of normality in returns with P-value in parenthesis. See section 2 for explanations on the calculation of r_t.

Table 2
Linearity Tests.

delay	Canada (p = 8)	France (p = 4)	Germany (p = 8)	Italy (p = 7)	Japan (p = 8)	U.K. (p = 8)	U.S. (p = 5)
1	8.3640	7.6684	8.2354	19.9021*	25.8158*	6.8823	5.5535
2	6.2199	8.9155	23.1717*	5.5993	11.7095	8.9045*	11.6317*
3	17.1471*	7.2303	9.1984	6.8174	10.7775	6.8397	3.0928
4	7.1109	8.9604*	11.6435	9.9699	6.0168	2.7186	6.8040

Notes: The asterisks indicate the maximum F-statistic for each delay (d) lag over the interval $1 \leq d \leq 4$. The selection of the maximum lag, p, of the linear AR model is made using the AIC statistic.

Table 3
Specification of the nonlinear model.

Country	Delay (d)	H_{04}: $\beta_4 = 0$	H_{03}: $\beta_3 = 0/\beta_4 = 0$	H_{04}: $\beta_2 = 0/\beta_3 = \beta_4 = 0$	Type of model
Canada	3	0.0035	0.0000	0.0000	LSTAR
France	4	0.0215	0.0000	0.0050	LSTAR
Germany	2	0.1745	0.0160	0.0000	ESTAR
Italy	1	0.0000	0.0000	0.0000	LSTAR
Japan	1	0.0010	0.0000	0.0000	LSTAR
U.K.	2	0.1035	0.0028	0.0000	ESTAR
U.S.	2	0.0436	0.0000	0.0000	LSTAR

Notes: The values for the nested tests H_{04}, H_{03}, H_{04} are P-values. The threshold value for the linearity tests and the specification of the STAR model is 0.05.

Table 4
Estimation of the Benchmark AR(p) model.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
ϕ_0	0.216	0.219	0.235	0.223	0.065	0.117	0.057
	(10.82)	(11.69)	(9.45)	(7.14)	(2.59)	(5.62)	(5.04)
ϕ_1	-0.614	-0.147	-0.271	-0.071	0.173	-0.147	-0.197
	(-10.56)	(-2.59)	(-4.10)	(-1.06)	(2.75)	(-2.70)	(-3.84)
ϕ_2	-0.443	-0.285	-0.312	-0.272	0.322	0.079	0.017
	(-6.73)	(-4.94)	(-4.78)	(-4.14)	(4.95)	(1.46)	(0.35)
ϕ_3	-0.411	-0.117	-0.119	-0.163	-0.033	-0.201	0.220
	(-6.46)	(-2.05)	(-1.81)	(-2.43)	(-0.48)	(-3.84)	(4.25)
ϕ_4	-0.258	-0.205	-0.082	-0.098	-0.477	-0.131	0.443
	(-4.34)	(-4.01)	(-1.27)	(-1.52)	(-7.66)	(-2.51)	(8.24)
ϕ_5	-0.407		-0.318	0.132	0.060	-0.046	0.176
	(-6.84)		(-5.00)	(2.08)	(0.99)	(-0.88)	(3.15)
ϕ_6	-0.360		-0.311	-0.224	0.253	-0.186	
	(-5.78)		(-4.54)	(-3.67)	(4.44)	(-3.62)	
ϕ_7	-0.141		-0.327	-0.126	-0.122	0.104	
	(-2.25)		(-4.72)	(-2.01)	(-2.03)	(2.00)	
ϕ_8	0.129		0.161		0.039	0.197	
	(2.34)		(2.50)		(0.66)	(3.84)	
R^2	0.421	0.135	0.322	0.187	0.313	0.179	0.192
SER	0.142	0.194	0.169	0.293	0.229	0.213	0.133

Notes: R^2 is the R-square of each model, SER is standard error of regression, and the t-statistics is reported in parenthesis.

Table 5
Estimates of STAR models.

	Canada (LSTAR)	France (LSTAR)	Germany (ESTAR)	Italy (LSTAR)	Japan (LSTAR)	U.K. (ESTAR)	U.S. (LSTAR)
ϕ_0	0.172	0.195	0.300	0.232	0.444	0.118	0.158
	(10.20)	(12.22)	(8.41)	(8.82)	(6.60)	(8.23)	(7.98)
ϕ_1	-0.344	-0.93		-0.555	0.561		-0.198
	(-6.18)	(-1.96)		(-6.14)	(6.43)		(-4.07)
ϕ_2		-0.124		-0.430	-0.373		0.109
		(-1.78)		(-10.81)	(-1.60)		(1.07)
ϕ_3	-0.417		-0.589	-0.266		-0.257	
	(-4.41)		(-7.95)	(-5.21)		(-4.86)	
ϕ_4	-0.202		-1.112		-0.543		-0.143
	(-3.62)		(-7.94)		(-10.93)		(-1.63)
ϕ_5	-0.554		-0.971				
	(-11.52)		(-7.32)				
ϕ_6	-0.452		-0.757	-0.384	0.171	-0.310	
	(-8.29)		(-6.32)	(-10.61)	(3.33)	(-5.83)	
ϕ_7	-0.390		-0.740	-0.350	-0.159		
	(-4.82)		(-4.35)	(-11.01)	(-2.71)		
ϕ_8			0.197		0.772	0.323	
			(4.56)		(4.48)	(6.23)	
ρ_0	0.550	0.483	-0.032	0.315	-0.445	-0.029	-0.156
	(2.27)	(5.13)	(-0.57)	(3.84)	(-5.01)	(-0.52)	(-4.21)
ρ_1	-1.570		-0.378			-0.333	0.096
	(-3.75)		(-4.56)			(-3.16)	(1.29)
ρ_2	-2.234	-0.694	-0.422		0.666		0.310
	(-3.99)	(-5.32)	(-5.00)		(2.75)		(1.90)
ρ_3	-1.526	-0.638	0.636				0.257
	(-2.45)	(-5.30)	(5.43)				(5.86)
ρ_4	-0.776	-1.343	1.298	-0.662			0.510
	(-3.86)	(-6.85)	(7.68)	(-5.54)			(3.31)
ρ_5			0.641			-0.172	
			(4.38)			(-1.73)	
ρ_6			0.383	0.321		0.843	
			(2.31)	(2.70)		(4.44)	
ρ_7	0.572		0.712	0.770		0.720	
	(2.36)		(3.94)	(4.19)		(3.02)	
ρ_8					-0.825	-0.515	
					(-4.27)	(-2.29)	
γ	2.278	5.584	7.129	15.873	4.612	1.970	11.628
	(4.11)	(2.01)	(4.19)	(1.02)	(2.73)	(3.43)	(2.01)
c	0.241	0.292	-0.013	-0.169	-0.217	0.100	0.029
	(4.89)	(13.59)	(-0.90)	(-6.50)	(-11.49)	(3.58)	(2.22)
R^2	0.668	0.298	0.632	0.447	0.459	0.317	0.265
SER	0.109	0.176	0.127	0.244	0.204	0.195	0.125

Notes: R^2 is the R square of each model, SER is standard error of regression, and the *t*-statistics is reported in parenthesis.

Table 6
Characteristic roots in each regime.

Country	Regime	Most prominent roots	Modulus
Canada (LSTAR)	Lower regime (F=0)	1.234	1.234
		-0.638±0.407i, -0.263±0.76i	0.757, 0.804
		0.457±0.802i	0.923
	Upper regime (F=1)	-0.801, 0.258, 2.720	0.801, 0.258, 2.720
		-0.503±0.681i, 0.202±0.641i	0.846, 0.672
France (LSTAR)	Lower regime (F=0)	-0.308, 0.401	0.308, 0.401
	Upper regime (F=1)	-1.110, 1.432	1.110, 1.432
		-0.115±0.912i	0.919
Germany (ESTAR)	Middle regime (F=0)	0.208, 1.347	0.208, 1.347
		-0.845±0.366i, -0.245±0.938i	0.921, 0.969
		0.313±0.887i	0.941
	Outer regime (F=1)	0.585, 1.038	0.585, 1.038
		-0.415±0.676i, -0.713±0.300i	0.793, 0.774
		0.505±0.779i	0.929
Italy (LSTAR)	Lower regime (F=0)	1.265	1.265
		-0.664±0.258i, -0.291±0.789i	0.712, 0.840
		0.599±0.642i	0.879
	Upper regime (F=1)	-0.961, 0.719, 1.286	0.961, 0.719, 1.286
		-0.426±0.642i, 0.182±0.875i	0.771, 0.894
Japan (LSTAR)	Lower regime (F=0)	-1.089±0.230i, -0.349±0.894i	1.113, 0.960
		0.269±0.836i, 0.889±0.296i	0.879, 0.937
	Upper regime (F=1)	-0.272, 0.764	0.272, 0.764
		-0.770±0.211i, -0.170±0.997i	0.799, 1.011
		0.414±0.469i	0.625
UK (ESTAR)	Middle regime (F=0)	-0.747±0.278i, -0.426±0.867i	0.797, 0.966
		-0.368±0.806i, 0.805±0.211i	0.886, 0.832
	Outer regime (F=1)	-0.805, 0.231	0.805, 0.231
		-0.656±0.708i, 0.151±1.000i	0.965, 1.012
		0.959±0.407i	1.042
US (LSTAR)	Lower regime (F=0)	-0.531, 0.615	0.531, 0.615
		0.053±0.638i	0.641
	Upper regime (F=1)	-0.494±0.567i	0.752
		0.558±0.762i	0.945

Table 7
Out-of-Sample Forecasting Performances.

Countries	Statistics	1 Step	2 Step	3 Step
U.S.	RMSPE	0.8284	0.8384	0.8275
	MAPE	0.8181	0.8392	0.8271
Japan	RMSPE	0.9850	0.9803	1.0510
	MAPE	0.9955	0.9935	1.0508
Germany	RMSPE	0.8405	0.8364	0.8151
	MAPE	0.8277	0.8361	0.8091
U.K.	RMSPE	0.9963	0.9984	0.9973
	MAPE	0.9921	0.9946	0.9951
France	RMSPE	1.2584	1.2531	1.2651
	MAPE	1.3211	1.3043	1.3235
Italy	RMSPE	0.7767	0.7558	0.7375
	MAPE	0.8287	0.7851	0.7496
Canada	RMSPE	1.2854	1.2385	1.2199
	MAPE	1.4119	1.3534	1.3326

Notes: RMSPE and MAPE are the ratios of the RMSPE (and MAPE) for the STAR model over the RMSPE (and MAPE) for the linear AR model. Values smaller than one indicate superior forecasting performance for the STAR model relative to the benchmark linear model.

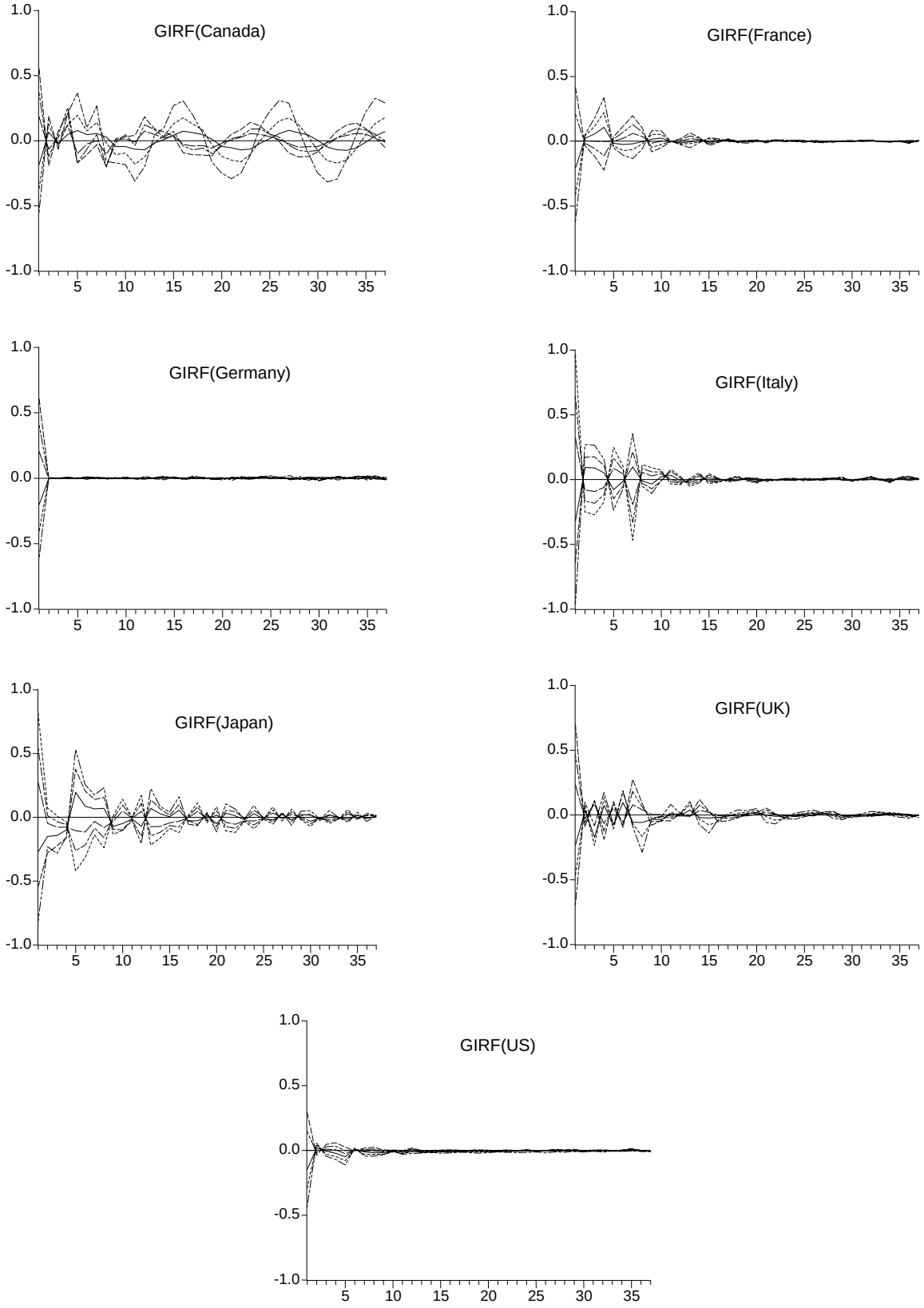
GIRF(Canada)
GIRF(France)
GIRF(Germany)
GIRF(Italy)
GIRF(Japan)
GIRF(UK)
GIRF(US)
1.0
0.5
0.0
-0.5
-1.0
5
10
15
20
25
30
35

Figure 1.